Some Perspectives of Fuller's Mathematics

An Undergraduate's Assessment by William R. Morrell, Yale College, Class of 1986

Editor's note: In proposing synergetics, Fuller's aim was to derive its generalizations from all of physical and metaphysical experience; thus his concepts invade a variety of academic disciplines as broad as philosophy and physics and psychology and mathematics. Within mathematics, synergetics employs methods analogous to topology, crystallography, and combinatorial geometry. But since the Synergetics books were published as trade books, they were largely ignored by the scholarly journals, and academics in general have felt no obligation to assess Fuller's philosophical and geometrical strategies in the context of their own disciplines. William Morrell's recent paper is a happy departure from this posture of neglect; it is a sympathetic attempt to relate synergetics to the world of conventional mathematics. That this first venture should come from an undergraduate would have been a particular gratification to Fuller.

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Mathematics is a troublesome discipline. It is difficult to define clearly where it fits into society and to understand what role it plays. Buckminster Fuller was displeased with the path humanity had chosen for math and science in general. Since the problems he found in math were not in its conclusions—which were deductively reasonable—but rather in its philosophical foundations, he tried to reformulate mathematics in a way that was consistent with his physical and metaphysical experience. *Synergetics* and *Synergetics 2* are his attempts to accomplish this task. He found it necessary to define a system of thought and method that would allow the math and geometry to be applicable not only to themselves but also to any field of thought. Math for Fuller is not a self-contained system based on axioms and divorced from reality, but rather the science of generalizations; math is generalized learning; it is the study of patterns in nature and universe, regardless of where they appear.

Accordingly he presented in *Synergetics* a system of analyzing system interactions; he avoided the "thingness of things" and "things as they are in themselves" and he examined only interactions, for interactions are all we can ever experience. We may never know how and what things really *are*, but we can speak of how they behave, both in and of themselves, and with respect to the rest of universe.

Fuller felt that the whole universe must be the starting point for his inquiry so that no inconsistencies would be created by neglected information. Fuller defined universe as "the aggregate of all humanity's consciously apprehended and communicated nonsimultaneous and only partially overlapping experiences" (*Synergetics* 301.10). Anything we can and ever will know is part of universe, and if we wish to understand how universe operates, we must keep in mind all of universe. Inherent in this formulation is that fact that we cannot know anything about anything outside Fuller's universe. We are necessarily inside universe, bound by our experience, and we must admit that we will never know why space, energy, and matter operate as they do. Fuller termed this the *a priori mystery*. Consequently, the only basis for finding "generalized principles" must be experience; we must observe how nature operates in order to be able to derive useful knowledge. This is why Fuller sought after "nature's coordinate system." He felt that the Greeks had erred in that their experiences were too local and had led to a flat-earth, orthogonal perception of space; discovering the way nature actually operated required thinking universally.

Another reason for thinking globally arises from the concept of synergy, the behavior of aggregates of parts as unpredicted by the constituents taken individually. If one studies deconstructively, examining events in isolation, one can only miss the behavior of these events in relation to universe. Also, we can never know anything about an object in isolation; we experience through interactions and relations, and so the only honest interpretation of a system must acknowledge synergy. "Intuition and mind apprehend that which is comprehensively between, and not of, the parts" (508.02).

In this attempt to set in order the facts of his experience, Fuller needed the concept of a system. A system is a subdivision of universe (400.011); it is a specific finite aggregate of events. Systems have an insideness and an outsideness, and they are polyhedral. Furthermore, systems constitute all experience; we experience a phenomenon by isolating it geometrically and in time. Fuller thought of math and geometry as the sciences of systems, the sciences of generalized interaction. One immediate consequence of this formulation is that any posited thought or specific event is irretrievably connected with its universal complement; unity is plural (501.13). Also, as a finite aggregate of events, a system may be represented by the closed configuration of the interactions of its parts and the interactions of these parts with the "outside," the universal complement. This is the reason for Fuller's use of vectors as the model for all interaction, for vectors indicate magnitude and direction, all that is necessary to specify an interaction.

Systems are dynamic; they continually interact with the rest of universe. It is natural to ask if there are systems which cohere in time. That such systems exist is not obvious a priori, but experience tells us that they do exist and these are called structures. Fuller defined a structure as a "self-stabilizing energy event complex" (600.02), "a locally regenerative pattern integrity of universe" (606.01). Structures are systems whose self-interaction tends to regenerate the system itself. What types of systems are these "pattern integrities"? Fuller posited that the only coherent systems are those which are triangulated. This seems reason enough to reject the Euclidean geometry-based mathematical institutions and search for a more realistic and honest-to-universe formulation. Triangulation seemed too fundamental to be absent in the foundations of any practical geometry.

Fuller felt that math and geometry should reflect the world he saw, which didn't include continuous surfaces, infinity, and dimensionless points, lines and planes, so he formulated synergetics without these abstract notions. Continuous surfaces were replaced by networks of vectors; infinity was rejected, as were points, lines and planes. Nature didn't have perfect spheres, it had polyhedra. Since math should be a generalization of reality

—of universe—it should reflect in its structure these experimentally demonstrable facts. Consequently Fuller modeled his geometry on triangulated patterns, the geometry nature used in the structure of matter (201.03). The tetrahedron, the octahedron, and the icosahedron were revealed to Fuller as the only regular triangular structures—the fundamental structure of nature. He reasoned that all other structures must be based upon these simplest structures. In coordination with his conception of universe and systems he used topology and vector geometry as the mathematical foundations of synergetics (201.01)—vector geometry for the reasons mentioned above and topology because it was capable of representing the behavior of polyhedral systems.

There is of course the question of nonregenerative systems, those which are essentially dynamic. Fuller attempted to break down the dynamic evolution of these systems into fundamental motions: spin, orbit, convergence and divergence, torque and precession. In structures, the forces of compression and tension balance the tendencies of the parts to converge or diverge. In orbits, precession balances convergence, divergence and torque. In trying to find out how and where these motions occur in universe Fuller utilized the concept of entropy. Any system moves in the path of least resistance in its evolution (305.02), and the natural flow of energy is toward greater entropy or disorder (330). This does not imply impending chaos, for one may harness the flow of energy in such a way as to create order, just as a sailboat may sail upwind, against the local flow of energy.

All of the above is part of synergetics, but only a small part. Fuller includes many applications and special-case realizations of the ramifications of these concepts in *Synergetics* and *Synergetics 2*. Among the more basic applications are Fuller's geodesic domes and tensegrity structures. These experimentally demonstrable realizations of the geometry were extremely important to Fuller, for they demonstrated that the method was valid. Domes are efficient and stable, simple and economical. They operate well in the environment. Tensegrity structures also signified much to Fuller because they are honest structures in which the types of interaction are clearly evident. In this sense tensegrity structures are the simplest systems.

These realizable artifacts were crucial to the completion of the methodology behind synergetics. Synergetics is experientially based, and should be experimentally applicable. One of Fuller's largest conflicts with the conventional institutional mathematics was that it didn't require itself to be directly connected with reality. In distinction from this, Fuller emphasizes the realizability of synergetics and its immediacy to reality in his definition, "Synergetics is the coordination of thought and physical action, the genesis of geometry, system, and structure" (261.01).

MATH IN SOCIETY

The results of Fuller's work in geometry and math, as presented in *Synergetics*, in no way exclude the results of conventional mathematics. The fundamental difference that appears is in the role Fuller saw math, geometry and science playing in society and in the relationship of these disciplines to empirical reality. The meaning of experience, learning and applications thereof is important here, as are the notions of abstraction and generalization.

In discussing the relationship of modern science to medieval science in "Modern Science, Metaphysics and Mathematics," Heidegger identifies the fundamental difference between the two as being the fact that modern science is "mathematical" (*Basic Writings*, New York: Harper & Row, 1977, p. 249). He uses the term mathematical according to the Greek *ta mathemata*: that which can be learned and, simultaneously, that which can be taught. He further identifies mathematics as ". . . a project of thingness which, as it were, skips over the things . . . the anticipation of the essence of things" through axioms (pp. 267–68). Accordingly, classical mathematics involves using experience to develop intuition into things as they are, and then proceeds synthetically toward abstraction of reality. It involves a priori assumptions inasmuch as it requires the primary intuition of the essence of space. This formulation is not staggeringly different from Fuller's synergetics. He identifies mathematics as "generalization, a third degree generalization that is a generalization of generalizations" (508.01). Math, for Fuller, is the science of generalizations; it is the abstraction of reality through generalizations of experience. The separation with conventional mathematics occurs in the point of abstraction where mathematicians divorce themselves from experience; after forming axioms—what they feel to be the statement of the fundamental essence of things—they leave the realm of reality. Synergetics is always considerate of experience and always in evolution in a plurality of directions, as opposed to the linear direction any axiomatic formulation must take (502.30).

Fuller was certainly not the first to require this absolute connection with observable reality. In *Prolegomena to Any Future Metaphysics*, Kant states, "Pure mathematics, and especially pure geometry, can have objective reality only on condition that they refer merely to the objects of sense. . . . It would be quite otherwise if the senses were so constituted as to represent objects as they are in themselves" (New York: Bobbs-Merrill, 1950, p. 34). We can know of universe only by way of our senses and extensions of them, so we can say nothing about the essence of things.

Inasmuch as science is the attempt to set in order the facts of experience (161), and inasmuch as mathematics is the science of pattern in experience through generalization (505.51), Fuller felt that the only self-consistent way of examining the universe was one in which the realms of experience, thought and action were interconnected through the "pyramid of generalizations." The only way relevancy and accuracy could be established in thought was by never forsaking the connection between thought and experience.

Having recognized that this condition on the development of mathematics exists, one may ask how one proceeds to discover these generalizations. Conventional mathematics uses abstract axioms and proceeds by deduction towards any conclusions which might be obtained. It is possible, then, for conventional math to stray far from experience if these axioms are incomplete or partially incorrect. Synergetics replaces these axioms with observed generalized principles. The thought progression is achieved by induction through intuition—where intuition arises from experience. In this sense Fuller regards conventional math as having begun with oversight (502.30). Fuller regards proper abstraction as reality, and inadequate abstraction as irrelevant. The only relevant abstractions are those which are consistent with reality (220.11). Generalizations, and particularly math, must arise through experience to be relevant. The obvious question is: relevant to what? Math should be relevant to the continuing operation of humanity.

Another of Fuller's motivations for rejecting conventional mathematics was that it had become too much of an institution. The extensive distancing of math and abstract science "produced the seemingly unbridgeable social chasm between the humanities and the sciences" (203.08). The mathematics institution was elitist; it had removed itself too far from reality for humanity as a whole to make use of it. Axiomatic thinking had resulted in specialization: each discipline needed its own axioms, for any axiomatic system must necessarily leave unanswerable questions, as Godel proved. This specialization had resulted in departmentalized thinking. Fuller promoted thinking synergetically, searching for all diverse information and all possible ramifications.

The key to Fuller's formulation of math lies in its applicability. He sought to derive pure principles because they, and only they, are universally usable (220.03). Generalized principles are inherently comprehensible and realizable. The realization is the necessary completion of the process; it is the teleological completion of abstract math. Math takes reality, abstracts it into principles, and finally uses those principles to act. It is a closed cyclic process between the physical and the metaphysical. The application closes the loop. Math, as a sort of generalized learning, should be immediate to life, not removed from it.

PERSPECTIVE

Fuller's philosophy regarding math reflects his social philosophy. Cerebration and abstraction are relevant only in so far as they are useful, applicable. "Mathematics is the science of structure and pattern in general" (MIT math department quote, 606.01). As such, math must not be limited by its own structure into considering only specific branches of reality; it must not axiomatize. If a truth of a pattern of behavior is discovered, it must hold everywhere. Reality, if regarded to be things and events as they really are, becomes meaningless, for it must embrace the *a priori mystery*.

The formulation Fuller arrived at in *Synergetics* does have applicability to society, and to some of the directions modern science is taking today. Finite noncontinuous systems and their dynamics have been given much attention in the past few years, and are still poorly understood. It is found that even extremely simple ecological population models can exhibit extraordinarily complex behavior. The behavior of the system at any given point is well defined, but no long-term solution exists; the dynamics exhibit "deterministic randomness." Another related field is that of fractal geometry. This is the study of recursively defined geometrical objects which, when analyzed, appear to have nonintegral dimensions, i.e., more than a line but less than an area. The objects of study frequently exhibit structure and pattern strikingly similar to that found in nature, despite the fact that they are generated by extremely simple models. Indeed, one of the first pieces of work in this area was a discussion of why the coastline of Great Britain was infinite. This field has discovered a number of universal behaviors of systems, mostly based on the work of Mitchell Feigenbaum at Cornell. The prospects of the future in this noncontinuum formulation of systems behavior are numerous; it may have relevance in such elusive problems as turbulence, the plasma physics involved in hydrogen fusion, and even quantum physics.

It seems that there is indeed a future for synergetics. Continuum mechanics can work, but diverse fields where it inevitably fails are being found. Fuller's synergetics may be the path to finding the general case which reduces to the conventional understanding of the specific case limit, as quantum mechanics does in the macroscopic limit. Synergetics does not contradict any previous work; it merely chooses a different path, and there is always value in asking a question a new way.

We find synergetics to be more operationally defined than conventional mathematics, both in intention and in formulation. Fuller requires this immediacy to reality as a precondition to any abstraction or generalization. He presents us with a system of thought which, while not entirely new or unique, is at least more internally consistent and honest than some other conventional approaches. Its scope is impressive, for it never axiomatizes, and thus allows itself more freedom to cover many diverse aspects of universe. The philosophy behind the work is inseparable from the math and geometry itself, and also from Fuller's social philosophy. We find mathematics and geometry defined experientially and operationally because Fuller wanted to create a tool. Its value lies in its use, which to this point has not been extensive. Synergetics has potential, it seems, but it must be used if it is to serve humanity at all.