Some Angles From Physics
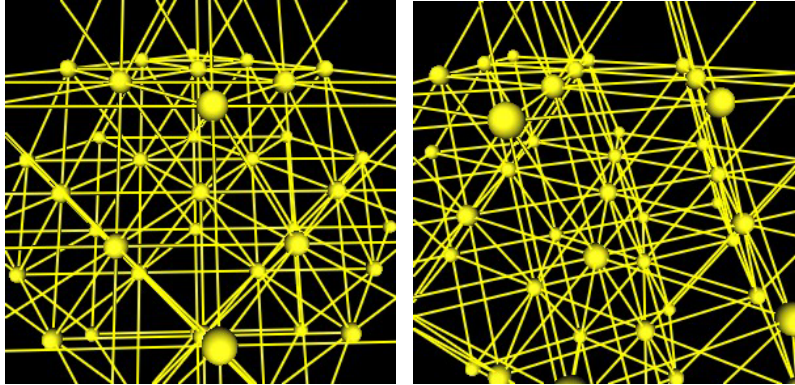# And Their Geometry

by

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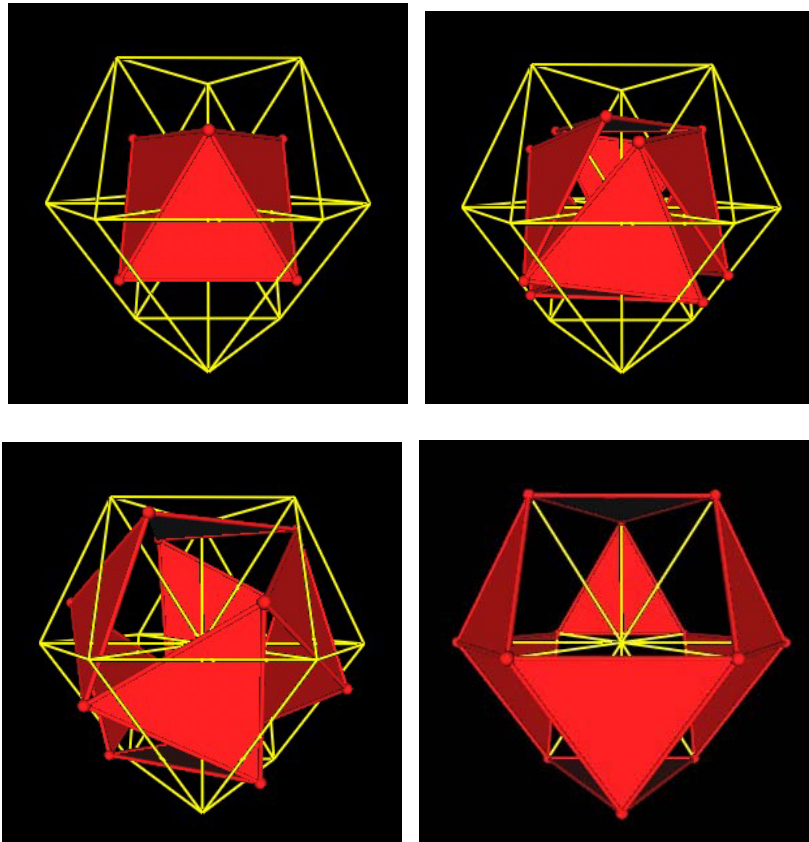**rwgray@rwgrayprojects.com**

09-30-2002

Here is an example of a FCC (Face Centered Cubic) lattice.
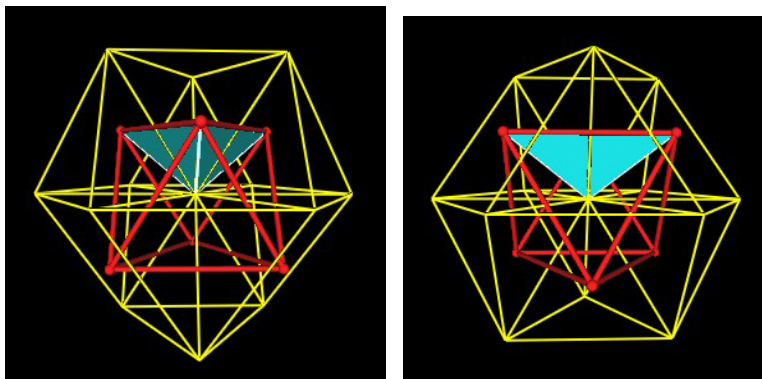

(See web version for animation.)

Within this lattice we place a Jitterbug such that when fully opened it matches the vertices of the FCC lattice.
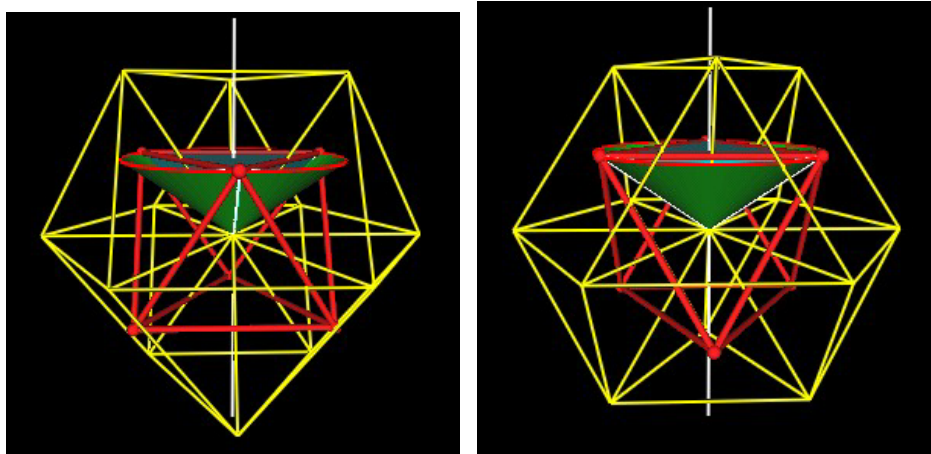

(See web version for animation.)

When closed, the Jitterbug (red) is in the shape of an Octahedron. When fully opened, the Jitterbug is in the shape of a cuboctahedron (a.k.a. Vector Equilibrium or just VE).

The Octahedron, having 8 triangular faces, may be divided into 8 irregular Tetrahedra. ("Irregular" here means that not all of the edges of the Tetrahedron are the same length.) These 8 irregular Tetrahedron are each defined by 3 vertices of the Octahedron plus the center of volume point. This defines the 4 vertices of the 1/8th Octahedra. One of these 1/8th of an Octahedron is shown in light blue below.

(See web version for animation.)

The rotation of this 1/8th Octahedron (an irregular Tetrahedron) defines a cone.
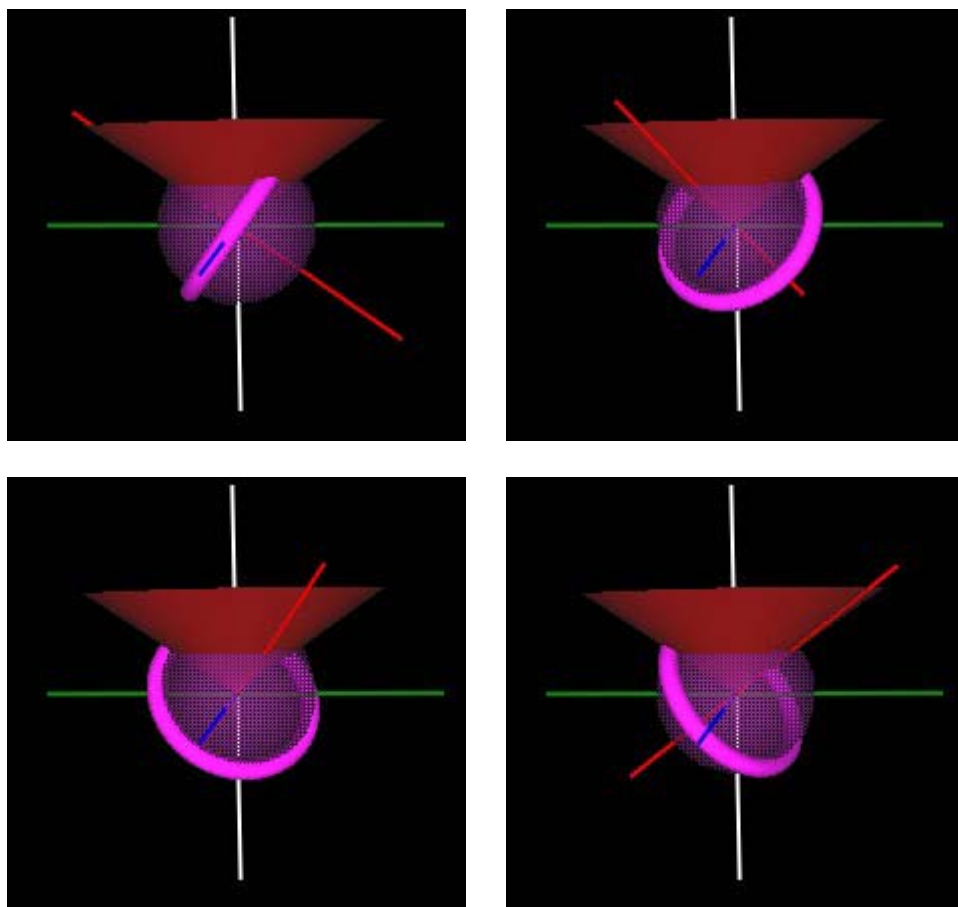
(See web version for animation.)

The cone angle is arccos(1/sqrt(3)) = 54.73561... degrees. (Some people may choose to call this the half-cone angle. It is the angle between the rotational symmetry axis (white line) and the surface of the cone.)

It is exactly the same angle as the precessional angle from Quantum Mechanics for a spin 1/2 particle in a magnetic field.

Here is a classical interpretation of the Quantum Mechanical, Spin Angular Momentum, Space Quantization, for a spin 1/2 particles (electron, proton) in a uniform magnetic field.

(See web version for animation.)

The applied magnetic field is along the z-axis (white line). The blue and green lines are the x and y axes. The red line is the spin angular momentum vector of the (pink/purple) particle. It makes an angle of arccos(1/sqrt(3)) = 54.73561... degrees with respect to the z-axis.

The equation for this angle from Quantum Mechanics is

$$\cos\left(\frac{m_j}{\sqrt{j(j+1)}}\right)$$

For a spin-1/2 particle, j = 1/2, m_j = (+-)1/2. This gives

$$\cos(\text{theta}) = (1/2) / \text{sqrt}(\ (1/2)(1/2 + 2/2)\ )$$
$$\cos(\text{theta}) = (1/2) / \text{sqrt}(\ (1/2)(3/2)\ )$$
$$\cos(\text{theta}) = (1/2) / [\ (1/2)\ \text{sqrt}(3)\ ]$$
$$\cos(\text{theta}) = 1 / \text{sqrt}(3)$$

which shows that theta = 54.73561...° and is the same geometric cone angle shown above.

This is also the cone angle for the cone of (force) equilibrium between 2 dipoles for the special case that the 2 dipoles always point in the same direction.

Consider 2 dipoles (magnets with North and South poles). These may be positioned so that one is at the coordinate origin while the other is some distance away on the y-axis.
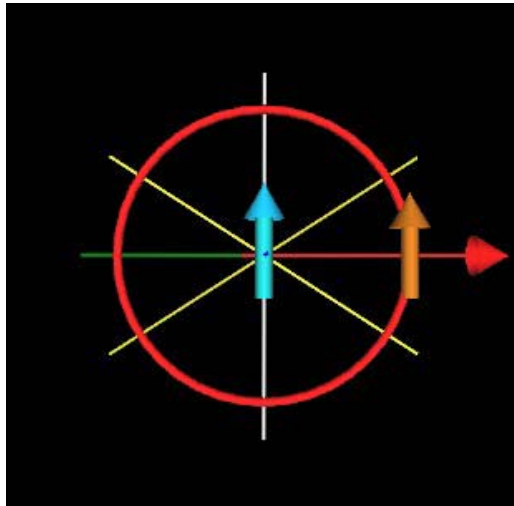

In the above figure, the z-axis is white running from top to bottom of the figure. The y-axis is green running from left to right of the figure. The x-axis is dark blue and is running out of the page.

A dipole (light blue) is at the coordinate origin. Another dipole (orange/light brown) is positioned some distance along the y-axis.

The force experienced by the orange/light brown dipole is represented by the red arrow/cone and by the red line of force connecting the two dipoles. The red

arrow/cone is pointing away from the light blue dipole meaning that the orange/light brown dipole experiences a repulsive radial force in this position.

If we advance the orange/light brown dipole around the light blue dipole, along the red circle, then there is a region for which the force experienced by the orange/light brown dipole will be attractive, toward the light blue dipole. This is represented by a dark blue arrow/cone at the end of the red line of force.
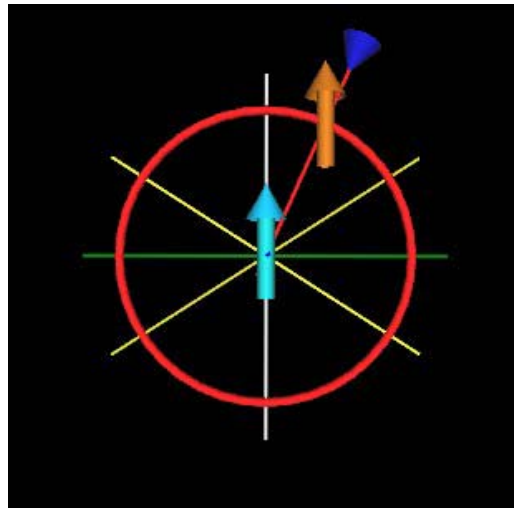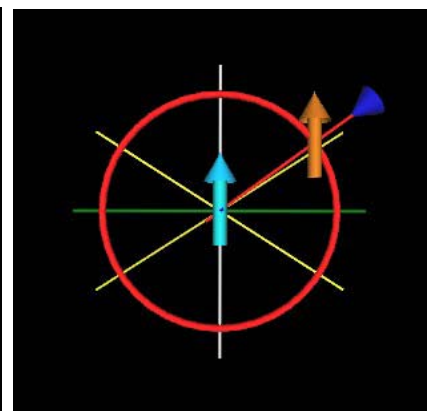

There is also drawn in these figures 2 lines of equilibrium (light yellow) which indicates where along the red circle the orange/light brown dipole will experience **no radial force** toward or away from the light blue dipole.
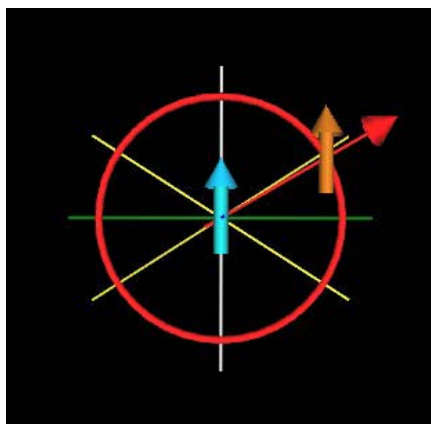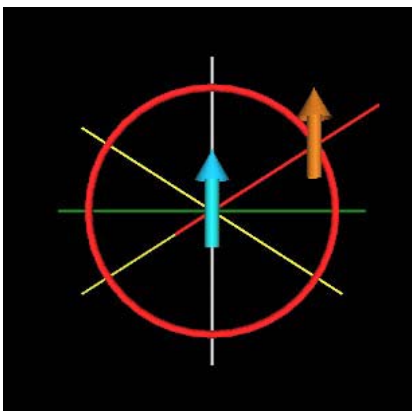

Repulsive Radial Force   Zero Radial Force   Attractive Radial Force

Two things to note: 1) The dipoles always point in the same direction (in the z-axis direction), and 2) In addition to the radial force being described here, the dipoles will experience a torque. This torque is not considered in these situations. Only the radial force between the 2 dipoles is being considered.

Here is the complete radial force cycle of the orange/light brown dipole around the center, light blue dipole.
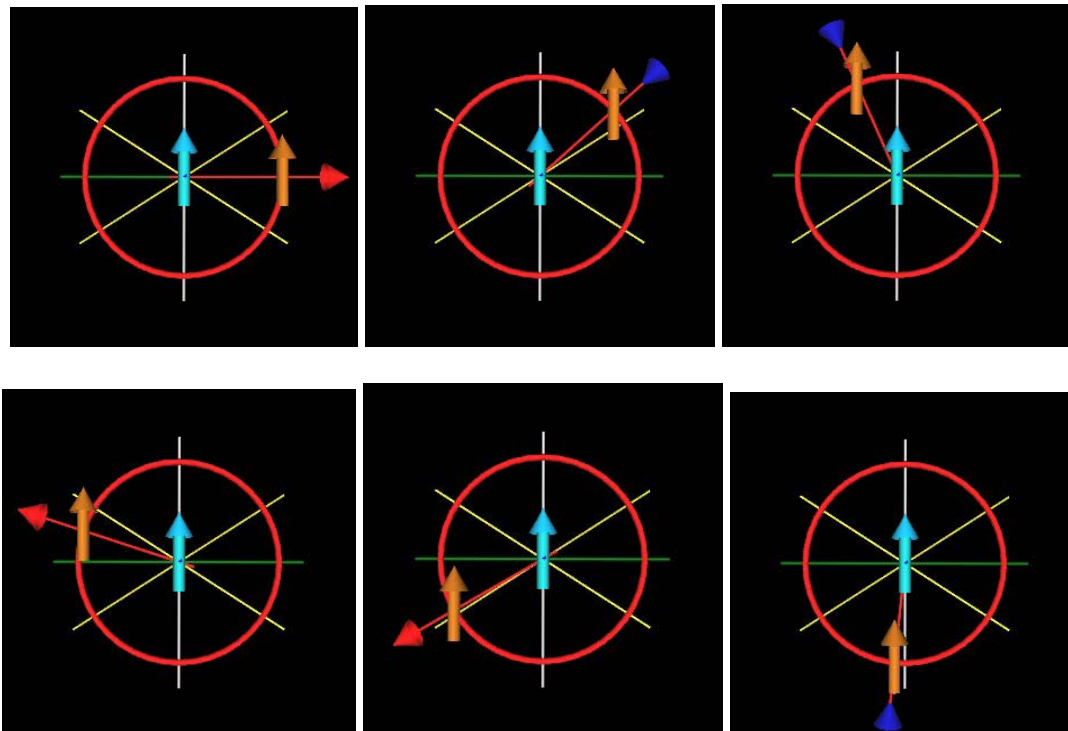
(See web version for animation.)

The light yellow lines make an angle of arccos(1/sqrt(3)) = 54.73561... degrees with respect to the z-axis.

If these light yellow lines are spun around the z-axis (see below for example) they would define 2 cones, one above the xy-plane and one below the xy-plane. The cone above the xy-plane is the same cone as illustrated above for the 1/8th Octahedron (the irregular Tetrahedron).

The equation for the radial component of the force between 2 dipoles is given by the equation

$$F = \frac{P_1 \, P_2}{r^4} \left(\cos\left(\varepsilon\right) - 3\cos\left(\alpha\right)\cos\left(\beta\right) \right)$$

where epsilon is the angle between the direction that dipole 1 is pointing in and the direction that dipole 2 is pointing in. Alpha is the angle between dipole 1 direction and the line connecting dipole 1 to dipole 2. Beta is the angle between dipole 2 direction and the line connecting dipole 1 to dipole 2.

For the special (artificial?) case of dipole 1 pointing in the same direction as dipole 2, we have

epsilon = 0 degrees

alpha = beta

This reduces the angular factor in the force equation to

(cos(0) - 3 cos(alpha)cos(alpha))

The radial force on these dipoles is then zero when this angular factor is zero. This is the case when
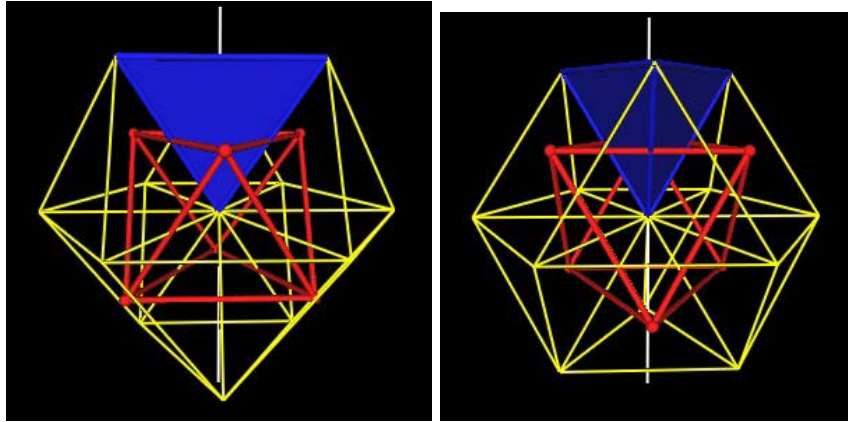
$$3 \cos^2(alpha) = \cos(0) = 1$$
$$\cos(alpha) = sqrt(1/3)$$

Which means alpha = 54.73561...°, the same geometric angle illustrated above.

Consider now one of the regular Tetrahedra within the (yellow) FCC lattice.

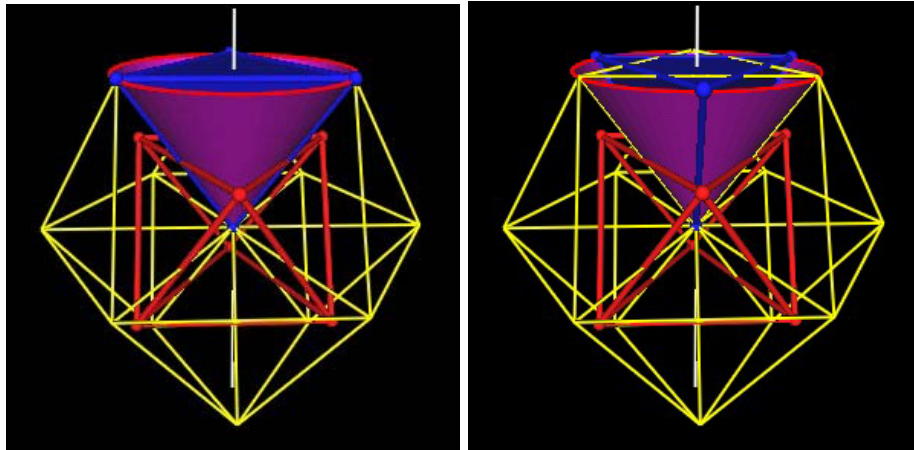(See web version for animation.)

The top triangle of this (blue) Tetrahedron is the same size as the triangular faces of the Octahedron Jitterbug (red).

The Jitterbug opens until these (red and blue) triangle faces match. (This was how we defined the size of the Jitterbug initially, above, so no surprise.)
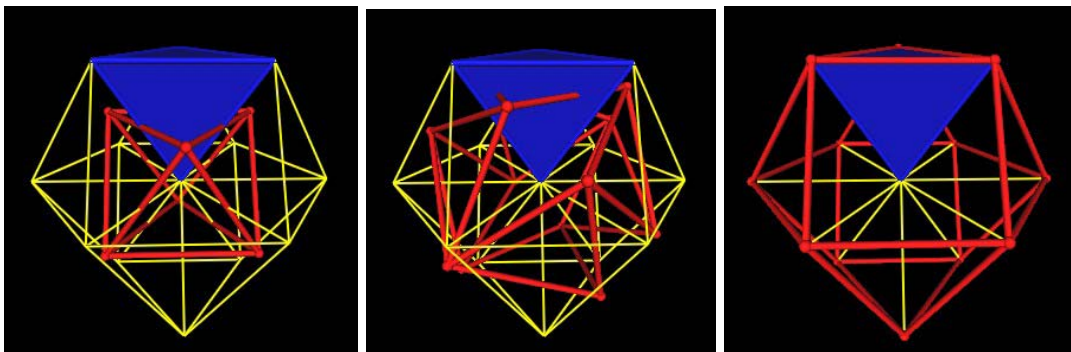
(See web version for animation.)

Spinning the (blue) Tetrahedron defines another cone.

(See web version for animation.)

The cone angle is the same as the cone of equilibrium (force equal zero) between 2 Amperian current elements for the special case that the 2 current elements always point in the same direction.

The cone angle is exactly arccos(sqrt(2/3)) = 35.264389... degrees.

Consider the following figure showing 2 Amperian current elements.
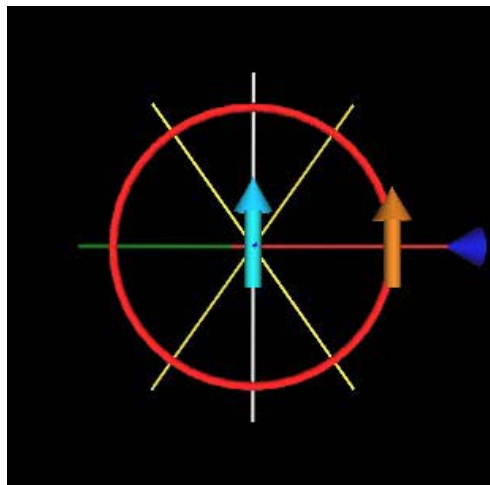
In this case, when the 2 Amperian Current Elements (ACE) are side-by-side and pointing in the same direction, the orange/light brown ACE will experience

an attractive force toward the light blue ACE, as indicated by the dark blue arrow/cone.

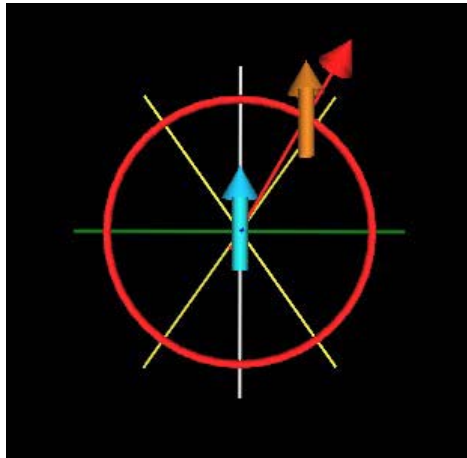As the orange/light brown ACE is moved around the light blue ACE, along the red circle, there will be regoins for which the force will be repulsive (indicated by a red arrow/cone).

The light yellow lines indicates where the **radial** component of the force is zero, and divides the attractive force regions from the repulsive force regions. (Again, only the radial component of the complete force is being considered here. There are also torque forces on the 2 ACEs which are not considered here making this an artificial arrangement.)

Here is the complete force cycle for 2 Amperian current elements (which are constrained to point in the same direction).
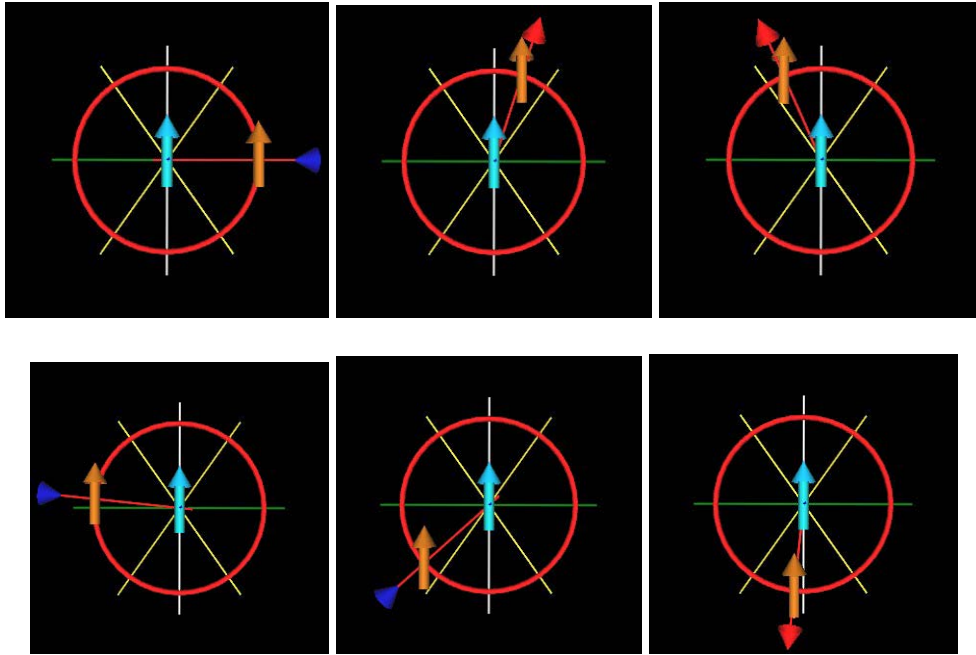
(See web version for animation.)

Note that these 2 light yellow lines of zero radial force (equilibrium lines) make an angle of exactly arccos(sqrt(2/3)) = 35.264389... degrees with respect to the z-axis.

If we now rotate these light yellow equilibrium lines around the z-axis, we define 2 cones of equilibrium (cones of zero radial force).
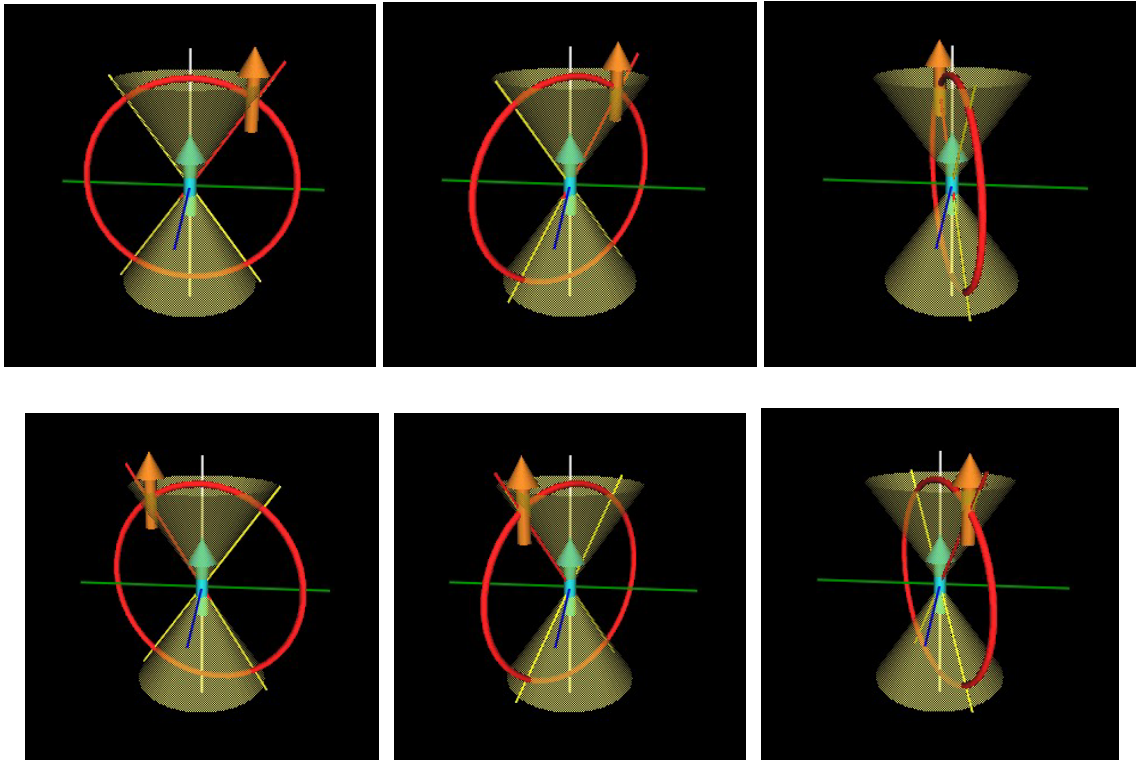
(See web version for animation.)

This is the same cone as that generated by the blue regular Tetrahedron of the VE mentioned above (and, of course, of the Tetrahedra in the FCC lattice).

The equation for the radial component of the force between 2 Amperian current elements is given by

$$\Delta F = \frac{I_1 \, dl_1 \, I_2 \, dl_2}{r^2} \left(2 \cos(\varepsilon) - 3 \cos(\alpha) \cos(\beta) \right)$$

Note that this is refered to as "Ampere's force law" and should not be confused with "Ampere's Law" which is one of Maxwell's equations.

As before, epsilon is the angle between the 2 dipole's directions. For the case that the 2 dipoles are pointing in the same direction, epsilon=0°. Alpha and beta are the angles between the dipole direction and the line connecting the 2

dipoles together. In this case the 2 dipoles are pointing in the same direction so we have alpha=beta. The angular factor of the force equation is then

$$(2 \cos(0) - 3 \cos(alpha) \cos(alpha))$$

This factor is zero, and hence the radial force is zero, when

$$3 \cos^2(alpha) = 2$$
$$\cos(alpha) = sqrt(2/3)$$

Which gives alpha = 35.264389...°, the same geometric cone angle mentioned above.

Here we have one Quantum Mechanical phenomenon (Angular Momentum Quantization) and two different classical electrodynamic phenomena:

1. dipole interactions,
2. Amperian current element interactions,

which seem to exhibit a geometric coordination under certain constraints (direction of orientation).

Also note that

$$\arccos(\sqrt{2/3}) + \arccos(1/\sqrt{3}) = 90.0°$$
$$35.26438...° + 54.73561...° = 90.0°$$

It is interesting that these 2 cone angles occur in the same geometry, having the same cone vertex point, and are limits (min and max) of the Jitterbug dynamics.

Here we have both tetrahedra rotating together.
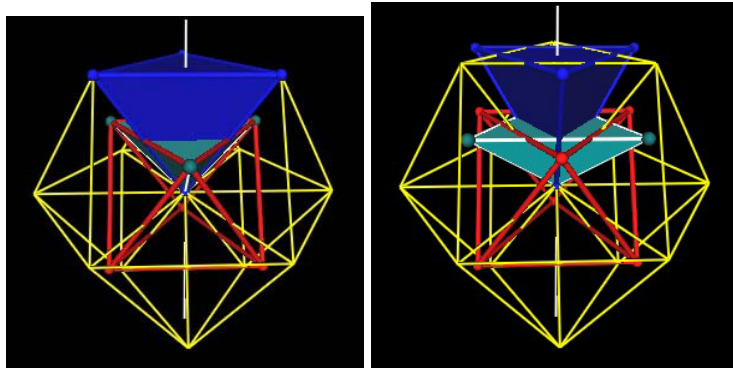
(See web version for animation.)

Defining their cones.

(See web version for animation.)

We now need to know a little bit more about FCC lattices.

Note that an orientation axis system (x-, y-, z-axis) can be defined for the Face Centered Cubic (FCC) lattice (the lattice of closest packing of spheres) such that all the vertices can be given integer valued (x, y, z) coordinates.
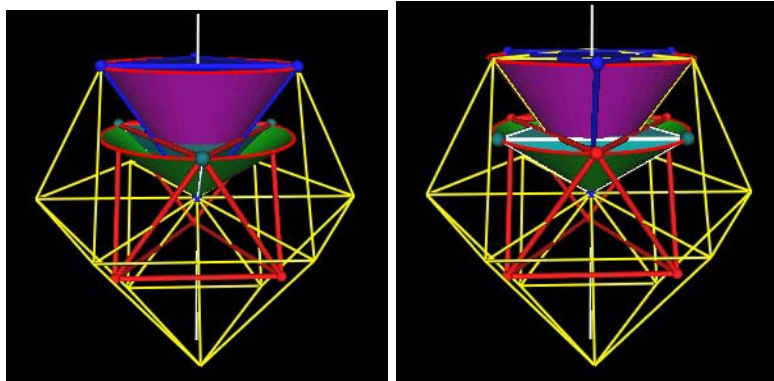
Further, the axes can be so oriented that the "logical even/odd sum" of the x, y, z coordinates is always even (or alternatively always odd, see below). The even/odd summation operation is summarized in the following table ("e"=even, "o"=odd):

(x, y, z)	Logical Sum Even/Odd
(e, e, e)	even
(e, e, o)	odd
(e, o, e)	odd
(e, o, o)	even
(o, e, e)	odd
(o, e, o)	even
(o, o, e)	even
(o, o, o)	odd

The FCC lattice for which all the vertex corrdinates are "logically summed" even includes the coordinate (0, 0, 0). That is, a lattice vertex will be located at (0, 0, 0).

An example of this is the yellow lattice shown previously. The Cuboctahedron (VE) was taken as a small part of the FCC lattice. Note that it has a vertex at its center of volume. We used this center of volume vertex (corrdinate (0,0,0)) as the apex point of the cones.

Another FCC lattice can be defined in which the vertex coordinates (x, y, z) are all integers and "logically summed" odd. For this (pink) "odd" FCC lattice,

there is no vertex at the coordinate origin (0,0,0). Note that in this case, the coordinate origin (0,0,0) occurs at the center of volume of an Octahedron.
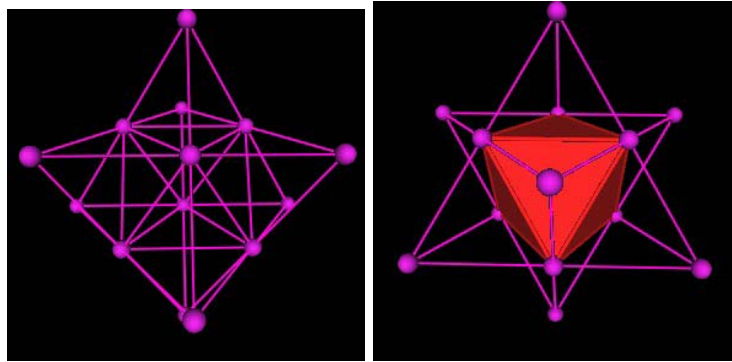
(See web version for animation.)

The Jitterbug motion occures between these two "even" (yellow) and "odd" (pink) FCC lattices.
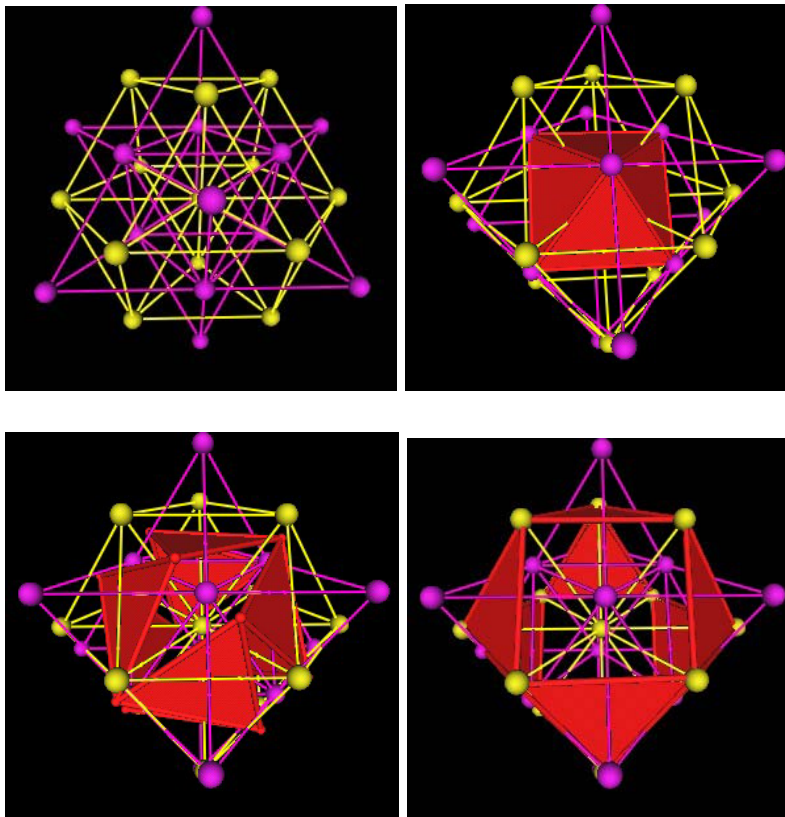
(See web version for animation.)

In the Electrodynamic theory of Myron Evans, as used by Tom Bearden to explain how his group's Motionless Electrical Generator (MEG) device can produce more energy out than is put in, it is shown that the applied voltage induces a dipole moment in the material. The dipoles act as gates allowing energy to flow into the device.

Could it be that Universe consists of 2 different spaces, represented here as 2 different FCC lattices? Could it be that the induced dipoles allows the energy of one space to be transduced into energy in the other space? Is the Jitterbug the dynamics which allows this to happen?

Perhaps these 2 lattices are the space and anti-space of particle physics. Particles consist of some dynamic system of the yellow lattice while antiparticles are the same dynamic system only of the pink lattice.

References

The Quantum Mechanical angular momentum, space quantization is described in any introductory Modern Physics or Introduction to Quantum Mechanics text. For example:

- Raymond A. Serway, et. al., Modern Physics, Harcourt Pub., August 1996
- S. B. Palmer, Quantum Physics, Gordon and Breach Science Publishers, May 1999

And many others....

For Ampere's force equation between 2 Ampere current elements see

- Peter Graneau, Neal Graneau, Newtonian Electrodynamics, World Scientific Pub Co, June 1996
- Peter Graneau, Ampere-Neumann Electrodynamics of Metals, 2nd edition, (Hadronic Press Monographs in Theoretical Physics), Hadronic Press, August 1994
- Peter Graneau, Newton Versus Einstein, Carlton Press, December 1992

These references also describe why an Amperian current element is not to be considered to be the same as electron drift currents.

These authors also point out that the Ampere force equation was not liked by some researchers of that time because there was no reason for the seemingly arbitrary angular arrangement resulting in a zero radial force. This is the

35.264389...° angle illustrated above. I wonder why researchers of that time complained about this angle but not the corresponding zero force angle in the dipole force equation.

For the dipole force equation, see any of the "standard" Electrodynamic texts. For example:

- John David Jackson, Classical Electrodynamics, 3rd edition, John Wiley & Sons, August 1998
- David J. Griffiths, Introduction to Electrodynamics, 3rd Edition, Prentice Hall, December 30, 1998

Curiously, and as the Graneau references point out, Ampere's force equation is not to be found in these or other "standard" Electrodynamic texts, yet Ampere's force equation is known to be experimentaly valid.

An introduction to Tom Bearden, et. al. Motionless Electric Generator can be found in the following references:

- P. K. Anastasovski, et. al., "EXPLANATION OF THE MOTIONLESS ELECTROMAGNETIC GENERATOR WITH 0(3) ELECTRODYNAMICS", Foundations of Physics Letters, Vol. 14., No. 1, 2001
- T. E. Bearden, "Extracting and Using Electromagnetic Energy from the Active Vacuum," in M.W. Evans (ed.), Contemporary Optics and Electrodynamics, Wylie, 2001, 3 Vols., comprising a Special Topic issue as vol. 114, I. Prigogine and S. A. Rice (series eds.), Advances in Chemical Physics, Wylie.
- See also Tom Bearden's website: http://www.cheniere.org/toc.html.

Myron Evans has published many papers on higher guage symmetry electrodynamics. Here are just a couple of references.

- Myron Evans, "O(3) Electrodynamics," Contemporary Optics and Electrodynamics, M. W. Evans (Ed.), special issue of I. Prigogine and S. A. Rice (series Eds.), Advances in Chemical Physics, Wiley, New York, 2001, vol. 114(2)
- See also the AIAS web site at http://www.aias.us.

Most of this material, with different graphics was originally presented on my web page at www.rwgrayprojects.com/Universe/ConeAngle/coneangle.html.