Basic Calculations Of The Wave Structure Of Matter Theory: Relativity Relations

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An outline of this material is provided in Milo Wolff's book Exploring The Physics Of The Unknown Universe, 2nd edition, 1994, pp. 238-245 and in the paper "Relativistic Mass Increase And Doppler Shift Without Special Relativity", Galilean Electrodynamics, Vol. 9, No. 6, pp. 117-120, Nov./Dec. 1998. Any errors in these calculations or errors in interpretation with respect to WSM are my own.

First, a review of wavelength and frequency shifts of waves in a classical medium.

Consider a source of waves in a medium. Assume the source is stationary with respect to the wave medium. The speed of the wave through the medium will be denoted by v_{wave}. The wavelength of the wave will be denoted by λ_0. Then the frequency of the wave is given by

$$f_0 = \frac{v_{wave}}{\lambda_0} \tag{1}$$

Now consider waves from a source that is moving (speed u_S) relative to the wave medium. The wavelength in front of the moving source will have a shortened wavelength given by

$$\lambda_A = \lambda_0 \left(1 - \frac{u_S}{v_{wave}}\right) \tag{2}$$

and the wavelength behind the moving source will be lengthened and given by

$$\lambda_B = \lambda_0 \left(1 + \frac{u_S}{v_{wave}}\right) \tag{3}$$

Note that the speed of the waves (v_{wave}) in the medium depends only on the properties of the medium and not on the speed of the source (u_S) through the medium. This means that when the receiver is at rest with respect to the wave medium and the source is moving toward the receiver (u_S), the frequency detected by the receiver will be

$$f' = \frac{v_{wave}}{\lambda_A} = \frac{v_{wave}}{\lambda_0\left(1 - \frac{u_S}{v_{wave}}\right)} = f_0 \frac{1}{\left(1 - \frac{u_S}{v_{wave}}\right)} \qquad (4)$$

Now consider the case of the source at rest relative to the wave medium and the receiver moving (speed u_R) relative to the wave medium. In this case there is no change to the wavelength of the wave in the medium because, again, the source is at rest with respect to the wave medium.

However, there is a frequency increase given by

$$f'' = f_0\left(1 + \frac{u_R}{v_{wave}}\right) \qquad (5)$$

If the receiver were moving away from the source then the frequency measured by the receiver would be

$$f'' = f_0\left(1 - \frac{u_R}{v_{wave}}\right) \qquad (6)$$

We now postulate that for waves in the "vacuum of space" (such as quantum waves or electromagnetic waves) the wave media is not detectable. (Some people would say that there is no wave medium.) We can not, then, base the speed of the wave source or the wave detector on knowing their relative speed with respect to the wave medium. But since one of either the source or the receiver was always at rest with respect to the medium in the above equations, we can use these equations with the relative speed between the source and the detector, without referring to the wave medium.

We define $c = v_{wave} =$ speed of light through vacuum, $u_{REL} = u_R = u_S =$ the relative speed between source and detector. We then write (4) and (5) as

$$f' = f_0 \frac{1}{\left(1 - \frac{u_{REL}}{c}\right)} \qquad (7)$$

$$f'' = f_0 \left(1 + \frac{u_{REL}}{c}\right) \qquad (8)$$

Since we can not tell which (the source or the receiver) is moving, the physics should be the same in either situation and we should have $f' = f''$. But this is not the case. We therefore seek a correction factor that scales these frequency changes such that

$$\frac{f'}{\gamma} = \gamma \ f'' \qquad (9)$$

Then

$$f_0 \frac{1}{\gamma \left(1 - \frac{u_{REL}}{c}\right)} = f_0 \ \gamma \left(1 + \frac{u_{REL}}{c}\right) \qquad (10)$$

$$\gamma^2 = \frac{1}{\left(1 - \frac{u_{REL}}{c}\right)\left(1 + \frac{u_{REL}}{c}\right)} \qquad (11)$$

$$\gamma^2 = \frac{1}{\left(1 - \frac{u_{REL}}{c} + \frac{u_{REL}}{c} - \frac{u_{REL}^2}{c^2}\right)} \qquad (12)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u_{REL}^2}{c^2}}} \qquad (13)$$

which is the relativistic Lorentz correction factor of Special Relativity Theory. With

$$\beta = \frac{u_{REL}}{c} \qquad (14)$$

this is written as

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \qquad (15)$$

We can now write (10) as

$$f_0 \frac{1}{\gamma\left(1-\frac{u_{REL}}{c}\right)} = f_0 \frac{1}{D} \qquad (16)$$

$$f_0\,\gamma\left(1+\frac{u_{REL}}{c}\right) = f_0 \frac{1}{D} \qquad (17)$$

We therefore have the relativistic Doppler frequency and wavelength correction factors

$$D = \gamma\left(1-\frac{u_{REL}}{c}\right) = \gamma\,(1-\beta) \qquad (18)$$

$$\frac{1}{D} = \gamma\left(1+\frac{u_{REL}}{c}\right) = \gamma\,(1+\beta) \qquad (19)$$

For the case of the WSM quantum waves, we first make the following observations:

1) The IN- and OUT-waves associated with a "particle" are assumed to be described (or represented) mathematically by spherically symmetric wave functions. (See: "Basic Calculations Of The Wave Structure Of Matter Theory: Particle & Anti-Particle Waves".)
2) Now assume that there are two particles "A" and "B" and that you (as an imagined measuring device) are particle "A".
3) You can only know about the ***relative motion*** between you ("A") and "B". You can not say "B is **moving**" or even "B is **moving** closer to me." You can only say that the **relative distance between you and "B"** is increasing or decreasing. There is nothing about which ("A", "B", both?) is doing the actual moving.
4) You are "A". You can only "know" or "see" or "measure" "B's" IN- and OUT-waves at "A" that are traveling along the line **between** you and "B". You ***don't have any information*** about any other part of "B's" IN- and OUT-waves. To

know or measure something other than between “A” and “B” would require a different vantage point. You don't know anything about “the other side” of B.

5) So, given (1) above, we ***represent mathematically*** an abstraction of the complete picture of “B” and of what is happening by spherically symmetric wave functions ***without any way*** to measure or to “really know”. There is no way of measuring anything else because that would require, again, ***different*** vantage points (frames of reference) and you (“A”) only have ***between-ness*** of the “A” and “B” portion of the total situation. We use the assumed mathematical representation to fill in the rest.
6) That is why we can apply the Doppler factors to the wave number “*k*” in the wave functions without regard to all *other* directions (requiring different frames of reference). We select the mathematical model to be spherically symmetric based on one point of view (frame of reference) which is all we ever have (a between-ness, never an omni-directional, multi-vantage point, spherical view) and we abstract that “local” knowledge to the “whole”.

So, consider the WSM particle wave function:

$$\psi_P(r,t) = \psi_{IN} - \psi_{OUT} = A_1 \frac{e^{(ik_1ct + ik_1r)}}{r} - A_3 \frac{e^{(ik_3ct - ik_3r)}}{r} \tag{20}$$

We first assume that the IN- and OUT- wave amplitudes are the same and that the two wave numbers are the same

$$\psi_P(r,t) = A \frac{e^{(ikct + ikr)}}{r} - A \frac{e^{(ikct - ikr)}}{r} \tag{21}$$

The wave number k is the inverse of the wavelength $k = \frac{2\pi}{\lambda}$. This gives

$kc = \frac{2\pi c}{\lambda} = \omega$, the angular frequency of the wave.

We wish to know how this (“B”) particle’s wave properties ***appear*** to change from the point of view of another (“A”) particle/observer/measurement-device when there is a relative motion between them. We pick the case that the *relative distance between* "A" *and* "B" *is decreasing*. In that case the IN-wave of "B", as measured by "A" as the IN-wave passes between "A" and "B", will appear to be red shifted (to a longer wave length) and B's OUT-wave, as measured by "A" between "A" and "B", will appear to be blue shifted (to a shorter wave length). We now look to see how to apply the relativistic Doppler factors

$$1/D = \gamma(1+\beta) > 1 \tag{22}$$

and

$$D = \gamma(1-\beta) < 1 \tag{23}$$

to increase B's IN-wave wavelength and to decrease B's OUT-wave wavelength. The wavelength is contained in the wave number parameter

$$k = \frac{2\pi}{\lambda} \tag{24}$$

Then to increase the IN-wave wavelength, we need

$$k_{IN} = \frac{2\pi}{\lambda(1/\mathrm{D})} = k\mathrm{D} \tag{25}$$

and to decrease the OUT-wave wavelength

$$k_{OUT} = \frac{2\pi}{\lambda\mathrm{D}} = k(1/\mathrm{D}) \tag{26}$$

We can write B's wave function as seen by "A" with this increase and decrease of the wavelengths as

$$\psi_{B-mb-A}(r,t) = A\,\frac{e^{(i\,ct+i\,r)\,kD}}{r} - A\,\frac{e^{(ict-i\,r)k/D}}{r} \tag{27}$$

where "*B-mb-A*" indicates that this is B's waves as measured by "A". Of course, "*r*" is part of the spherical coordinate system (r, θ, φ) centered at "B" and does not indicate a distance between "A" and "B". (B's waves as measured by "B" are spherically symmetric and given by (21) above.)

Substituting (22) and (23) into (27) gives

$$\psi_{B-mb-A}(r,t) = A\frac{e^{(ict+ir)k\gamma(1-\beta)}}{r} - A\frac{e^{(ict-ir)k\gamma(1+\beta)}}{r} \tag{28}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}\left(e^{(ict+ir)(k\gamma-k\gamma\beta)} - e^{(ict-ir)(k\gamma+k\gamma\beta)}\right) \tag{29}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}\left(e^{(i\,ctk\gamma - i\,ctk\gamma\beta + i\,rk\gamma - i\,rk\gamma\beta)} - e^{(ictk\gamma + ictk\gamma\beta - i\,rk\gamma - i\,rk\gamma\beta)}\right) \tag{30}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}\left(e^{(i\,ctk\gamma - i\,rk\gamma\beta)}e^{(-i\,ctk\gamma\beta + i\,rk\gamma)} - e^{(i\,ctk\gamma - i\,rk\gamma\beta)}e^{(ictk\gamma\beta - i\,rk\gamma)}\right) \tag{31}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}e^{(i\,ctk\gamma - i\,rk\gamma\beta)}\left(e^{(-i\,ctk\gamma\beta + i\,rk\gamma)} - e^{(ictk\gamma\beta - i\,rk\gamma)}\right) \tag{32}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}e^{(i\,ctk\gamma - i\,rk\gamma\beta)}\left(e^{i(-ctk\gamma\beta + rk\gamma)} - e^{-i(-ctk\gamma\beta + rk\gamma)}\right) \tag{33}$$

$$\psi_{B-mb-A}(r,t) = \frac{A}{r}e^{i\,k\gamma(ct - r\beta)}\;2i\sin\left(-ctk\gamma\beta + rk\gamma\right) \tag{34}$$

$$\psi_{B-mb-A}(r,t) = \frac{A'}{r}e^{i\,k\gamma(ct - r\beta)}\sin\left(ctk\gamma\beta - rk\gamma\right) \tag{35}$$

where $A' = -2i$ and we used $\sin(-x) = -\sin(x)$. This can be written as

$$\psi_{B-mb-A}(r,t) = \frac{A'}{r}e^{i\,k\gamma(ct - \beta r)}\sin\left(k\gamma\left(c\beta t - r\right)\right) \tag{36}$$

With $k = \dfrac{2\pi}{\lambda}$, $mc^2 = h\,f$ and $f = \dfrac{c}{\lambda}$ we have $mc = h/\lambda$ and so we can write

$\lambda = h/mc$. Here, h is Planck's constant

$$h \simeq 6.626\,068\,96 \times 10^{-34}\,\mathrm{J\cdot s} \simeq 4.135\,667\,33 \times 10^{-15}\ \mathrm{eV\cdot s} \tag{37}$$

Using these relations the exponent has the terms (taking out the 2π factor)

$$\gamma\frac{k}{2\pi}\beta = \gamma\frac{1}{\lambda}\frac{v}{c} = \gamma\frac{mc}{h}\frac{v}{c} = \frac{\gamma mv}{h} \tag{38}$$

$$\gamma \frac{k}{2\pi} c = \gamma \frac{1}{\lambda} c = \gamma \frac{mc}{h} c = \frac{\gamma mc^2}{h} \tag{39}$$

The sine function has the terms

$$\gamma \frac{k}{2\pi} = \gamma \frac{1}{\lambda} = \frac{\gamma mc}{h} \tag{40}$$

$$\gamma \frac{k}{2\pi} c\beta = \gamma \frac{1}{\lambda} c\beta = \gamma \frac{mc}{h} c\beta = \frac{\gamma mc^2 \beta}{h} \tag{41}$$

Wolff writes:

> This equation has the form of an exponential carrier wave modulated by a sinusoid. The surprising characteristics of the carrier wave are:
>
> wavelength = $h/\gamma \text{mv}$ = de Broglie wavelength
>
> frequency = $k\gamma c/2\pi = \gamma mc^2/h$ = mass-energy frequency
>
> velocity = c/β = phase velocity.
>
> The modulating sine function has:
>
> wavelength = $h/\gamma mc$ = Compton wavelength
>
> frequency = $\gamma mc^2 \beta/h = \beta(\text{mass frequency})$ =momentum frequency
>
> velocity = βc = v = relative velocity of two resonances.