A Quantum Module

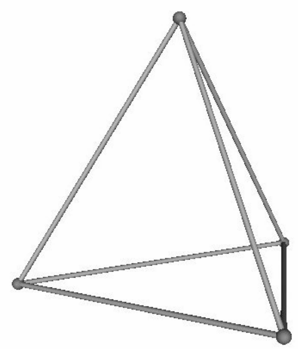

Figure 1 Regular Tetrahedron.

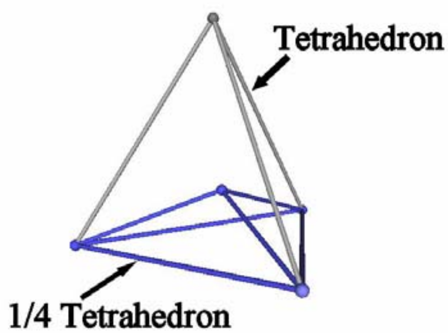

Figure 2 One of 4 Quarter Tetrahedra (blue) in the Tetrahedron.

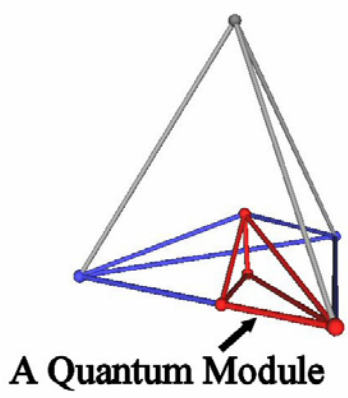

Figure 3 One of 6 A Quantum Modules (red) in the Quarter Tetrahedron.

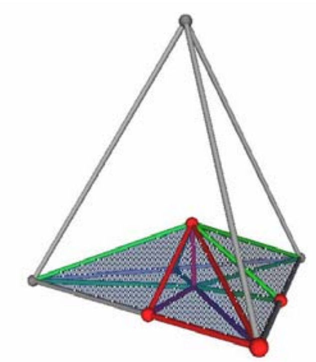

Figure 4 All 6 A Quantum Modules outline in one Quarter Tetrahedron.

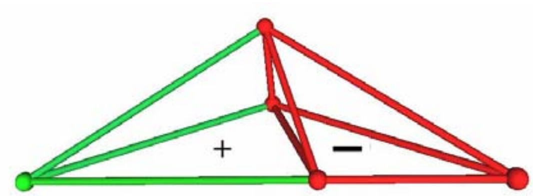

Figure 5 Positive (green) and Negative (red) A Quantum Modules.

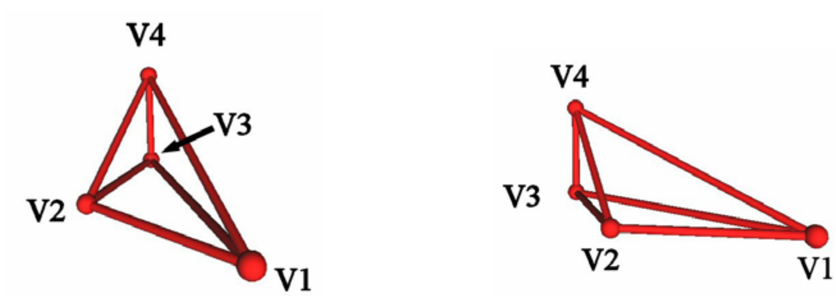

Figure 6 Vertex labeling of A Quantum Module.

Topology:

Vertices = 4

Edges = 6

Faces = 4 unequal triangles

Lengths:

EL ≡ Edge length of the regular Tetrahedron.

V1 ≡ Vertex of regular Tetrahedron.

V2 ≡ Mid-edge of regular Tetrahedron.

V3 ≡ Face center of regular Tetrahedron.

V4 ≡ Center of volume of regular Tetrahedron.

Edge Lengths:

$$V1.V2 = \frac{1}{2}\ EL \equiv \text{Half of the regular Tetrahedron's edge length.}$$

$$V1.V3 = \frac{1}{\sqrt{3}}\ EL \cong 0.577\ 350\ 269\ EL = DFV_T$$

$$V1.V4 = \frac{3}{2\sqrt{6}}\ EL \cong 0.612\ 372\ 436\ EL = DVV_T$$

$$V2.V3 = \frac{1}{2\sqrt{3}}\ EL \cong 0.288\ 675\ 135\ EL = DFE_T$$

$$V2.V4 = \frac{1}{2\sqrt{2}}\ EL \cong 0.353\ 553\ 391\ EL = DVE_T$$

$$V3.V4 = \frac{1}{2\sqrt{6}}\ EL \cong 0.204\ 124\ 145\ EL = DVF_T$$

Center of Face to Vertex:

$$DF(V1.V2.V3)V(V1) = \frac{\sqrt{13}}{6\sqrt{3}} \text{ EL} \cong 0.346\ 944\ 333 \text{ EL}$$

$$DF(V1.V2.V3)V(V2) = \frac{1}{3\sqrt{3}} \text{ EL} \cong 0.192\ 450\ 090 \text{ EL}$$

$$DF(V1.V2.V3)V(V3) = \frac{\sqrt{7}}{6\sqrt{3}} \text{ EL} \cong 0.254\ 587\ 539 \text{ EL}$$

$$DF(V1.V2.V4)V(V1) = \frac{1}{2\sqrt{2}} \text{ EL} \cong 0.353\ 553\ 391 \text{ EL}$$

$$DF(V1.V2.V4)V(V2) = \frac{1}{2\sqrt{6}} \text{ EL} \cong 0.204\ 124\ 145 \text{ EL}$$

$$DF(V1.V2.V4)V(V4) = \frac{1}{2\sqrt{3}} \text{ EL} \cong 0.288\ 675\ 135 \text{ EL}$$

$$DF(V1.V3.V4)V(V1) = \frac{\sqrt{11}}{6\sqrt{2}} \text{ EL} \cong 0.390\ 867\ 980 \text{ EL}$$

$$DF(V1.V3.V4)V(V3) = \frac{1}{2\sqrt{6}} \text{ EL} \cong 0.204\ 124\ 145 \text{ EL}$$

$$DF(V1.V3.V4)V(V4) = \frac{1}{3\sqrt{2}} \text{ EL} \cong 0.235\ 702\ 260 \text{ EL}$$

$$DF(V2.V3.V4)V(V2) = \frac{1}{2\sqrt{6}} \text{ EL} \cong 0.204\ 124\ 145 \text{ EL}$$

$$DF(V2.V3.V4)V(V3) = \frac{1}{6\sqrt{2}} \text{ EL} \cong 0.117\ 851\ 130 \text{ EL}$$

$$DF(V2.V3.V4)V(V4) = \frac{1}{6} \text{ EL} \cong 0.166\ 666\ 667 \text{ EL}$$

<u>Center of Face to Mid-edge:</u>

$$DF(V1.V2.V3)E(V1.V2) = \frac{\sqrt{7}}{12\sqrt{3}} \text{ EL} \cong 0.127\ 293\ 769 \text{ EL}$$

$$DF(V1.V2.V3)E(V1.V3) = \frac{1}{6\sqrt{3}} \text{ EL} \cong 0.096\ 225\ 045 \text{ EL}$$

$$DF(V1.V2.V3)E(V2.V3) = \frac{\sqrt{13}}{12\sqrt{3}} \text{ EL} \cong 0.173\ 472\ 167 \text{ EL}$$

$$DF(V1.V2.V4)E(V1.V2) = \frac{1}{4\sqrt{3}} \text{ EL} \cong 0.144\ 337\ 567 \text{ EL}$$

$$DF(V1.V2.V4)E(V1.V4) = \frac{1}{4\sqrt{6}} \text{ EL} \cong 0.102\ 062\ 073 \text{ EL}$$

$$DF(V1.V2.V4)E(V2.V4) = \frac{1}{4\sqrt{2}} \text{ EL} \cong 0.176\ 776\ 695 \text{ EL}$$

$$DF(V1.V3.V4)E(V1.V3) = \frac{1}{6\sqrt{2}} \text{ EL} \cong 0.117\ 851\ 130 \text{ EL}$$

$$DF(V1.V3.V4)E(V1.V4) = \frac{1}{4\sqrt{6}} \text{ EL} \cong 0.102\ 062\ 073 \text{ EL}$$

$$DF(V1.V3.V4)E(V3.V4) = \frac{\sqrt{11}}{12\sqrt{2}} \text{ EL} \cong 0.195\ 433\ 990 \text{ EL}$$

$$DF(V2.V3.V4)E(V2.V3) = \frac{1}{12} \text{ EL} \cong 0.833\ 333\ 333 \text{ EL}$$

$$DF(V2.V3.V4)E(V2.V4) = \frac{1}{12\sqrt{2}} \text{ EL} \cong 0.058\ 925\ 565 \text{ EL}$$

$$DF(V2.V3.V4)E(V3.V4) = \frac{1}{4\sqrt{6}} \text{ EL} \cong 0.102\ 062\ 073 \text{ EL}$$

<u>Center of Volume to Vertex:</u>

$$DVV(V1) = \frac{\sqrt{21}}{8\sqrt{2}} \text{ EL} \cong 0.405\ 046\ 294 \text{ EL}$$

$$DVV(V2) = \frac{\sqrt{5}}{8\sqrt{2}} \text{ EL} \cong 0.197\ 642\ 354 \text{ EL}$$

$$DVV(V3) = \frac{\sqrt{5}}{8\sqrt{2}} \text{ EL} \cong 0.197\ 642\ 354 \text{ EL}$$

$$DVV(V4) = \frac{\sqrt{23}}{8\sqrt{6}} \text{ EL} \cong 0.244\ 736\ 253 \text{ EL}$$

Center of Volume to Mid-edge:

$$DVE(V1.V2) = \frac{\sqrt{5}}{8\sqrt{2}}\ EL \cong 0.197\ 642\ 354\ EL$$

$$DVE(V1.V3) = \frac{\sqrt{7}}{8\sqrt{6}}\ EL \cong 0.135\ 015\ 431\ EL$$

$$DVE(V1.V4) = \frac{\sqrt{7}}{8\sqrt{6}}\ EL \cong 0.135\ 015\ 431\ EL$$

$$DVE(V2.V3) = \frac{\sqrt{7}}{8\sqrt{6}}\ EL \cong 0.135\ 015\ 431\ EL$$

$$DVE(V2.V4) = \frac{\sqrt{7}}{8\sqrt{6}}\ EL \cong 0.135\ 015\ 431\ EL$$

$$DVE(V3.V4) = \frac{\sqrt{5}}{8\sqrt{2}}\ EL \cong 0.197\ 642\ 354\ EL$$

Center of Volume to Face Center:

$$DVF(V1.V2.V3) = \frac{\sqrt{23}}{24\sqrt{6}}\ EL \cong 0.081\ 578\ 751\ EL$$

$$DVF(V1.V2.V4) = \frac{\sqrt{5}}{24\sqrt{2}}\ EL \cong 0.065\ 880\ 785\ EL$$

$$DVF(V1.V3.V4) = \frac{\sqrt{5}}{24\sqrt{2}}\ EL \cong 0.065\ 880\ 785\ EL$$

$$DVF(V2.V3.V4) = \frac{\sqrt{7}}{8\sqrt{6}}\ EL \cong 0.135\ 015\ 431\ EL$$

Areas:

$$\text{V1.V2.V3} = \frac{1}{8\sqrt{3}}\ \text{EL}^2 \cong 0.072\ 168\ 784\ \text{EL}^2$$

$$\text{V1.V2.V4} = \frac{1}{8\sqrt{2}}\ \text{EL}^2 \cong 0.088\ 388\ 348\ \text{EL}^2$$

$$\text{V1.V3.V4} = \frac{1}{12\sqrt{2}}\ \text{EL}^2 \cong 0.058\ 925\ 565\ \text{EL}^2$$

$$\text{V2.V3.V4} = \frac{1}{24\sqrt{2}}\ \text{EL}^2 \cong 0.029\ 462\ 783\ \text{EL}^2$$

$$\text{Total face area} = \frac{6+\sqrt{6}}{24\sqrt{2}}\ \text{EL}^2 \cong 0.248\ 945\ 479\ \text{EL}^2$$

$$= \frac{6+\sqrt{6}}{6\sqrt{2}}\ (\text{V1.V2})^2 \cong 0.995\ 781\ 916\ (\text{V1.V2})^2$$

Volume:

$$\text{Cubic measure volume equation} = \frac{1}{144\sqrt{2}}\ \text{EL}^3 \cong 0.004\ 910\ 464\ \text{EL}^3.$$

$$\text{Synergetics' Tetra-volume equation} = \frac{1}{24}\ \text{EL}^3 \cong 0.041\ 666\ 667\ \text{EL}^3$$

Angles:

Face Angles:

Sum of face angles = 720°

Face V1.V2.V3:

V2.V1.V3 = 30°

V1.V2.V3 = 90°

V1.V3.V2 = 60°

Face V1.V2.V4:

$$\text{V2.V1.V4} = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\,389\,683°$$

V1.V2.V4 = 90°

$$\text{V1.V4.V2} = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\,610\,317°$$

Face V1.V3.V4:

$$\text{V3.V1.V4} = \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 19.471\,220\,634°$$

V1.V3.V4 = 90°

$$\text{V1.V4.V3} = \arcsin\left(\frac{2\sqrt{2}}{3}\right) \cong 70.528\,779\,366°$$

Face V2.V3.V4:

$$\text{V3.V2.V4} = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

V2.V3.V4 = 90°

$$\text{V2.V4.V3} = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$$

Central Angles (identified by edge labels):

$$\text{V1.V2} = \arccos\left(\frac{-3}{\sqrt{105}}\right) \cong 107.023\ 866\ 185°$$

$$\text{V1.V3} = \arccos\left(\frac{-25}{3\sqrt{105}}\right) \cong 144.414\ 697\ 544°$$

$$\text{V1.V4} = \arccos\left(\frac{-29}{3\sqrt{161}}\right) \cong 139.626\ 683\ 085°$$

$$\text{V2.V3} = \arccos\left(\frac{-1}{15}\right) \cong 93.822\ 553\ 729°$$

$$\text{V2.V4} = \arccos\left(\frac{-\sqrt{15}}{3\sqrt{23}}\right) \cong 105.616\ 129\ 405°$$

$$\text{V3.V4} = \arccos\left(\frac{11\sqrt{3}}{3\sqrt{115}}\right) \cong 53.685\ 288\ 534°$$

Dihedral Angles (identified by edge labels):

$$V1.V2 = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

V1.V3 = 90°

V1.V4 = 60°

V2.V3 = 90°

V2.V4 = 90°

V3.V4 = 60°

Vertex Coordinates (X, Y, Z):

Positive A Quantum Module:

$$V1 = \left(\frac{-3}{8}, \frac{-1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) EL$$

$$\cong (-0.375,\ -0.144\ 337\ 567,\ -0.051\ 031\ 036)\ EL$$

$$V2 = \left(\frac{1}{8}, \frac{-1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) EL$$

$$\cong (0.125,\ -0.144\ 337\ 567,\ -0.051\ 031\ 036)\ EL$$

$$V3 = \left(\frac{1}{8}, \frac{1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) EL$$

$$\cong (0.125,\ 0.144\ 337\ 567,\ -0.051\ 031\ 036)\ EL$$

$$V4 = \left(\frac{1}{8}, \frac{1}{4\sqrt{3}}, \frac{3}{8\sqrt{6}}\right) EL$$

$$\cong (0.125,\ 0.144\ 337\ 567,\ 0.153\ 093\ 109)\ EL$$

Negative A Quantum Module:

$$V1 = \left(\frac{3}{8}, \frac{-1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}} \right) EL$$

$\cong$ (0.375, -0.144 337 567, -0.051 031 036) EL

$$V2 = \left(\frac{-1}{8}, \frac{-1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}} \right) EL$$

$\cong$ (-0.125, -0.144 337 567, -0.051 031 036) EL

$$V3 = \left(\frac{-1}{8}, \frac{1}{4\sqrt{3}}, \frac{-1}{8\sqrt{6}} \right) EL$$

$\cong$ (-0.125, 0.144 337 567, -0.051 031 036) EL

$$V4 = \left(\frac{-1}{8}, \frac{1}{4\sqrt{3}}, \frac{3}{8\sqrt{6}} \right) EL$$

$\cong$ (-0.125, 0.144 337 567, 0.153 093 109) EL

Unfolded Vertex Coordinates (X, Y):

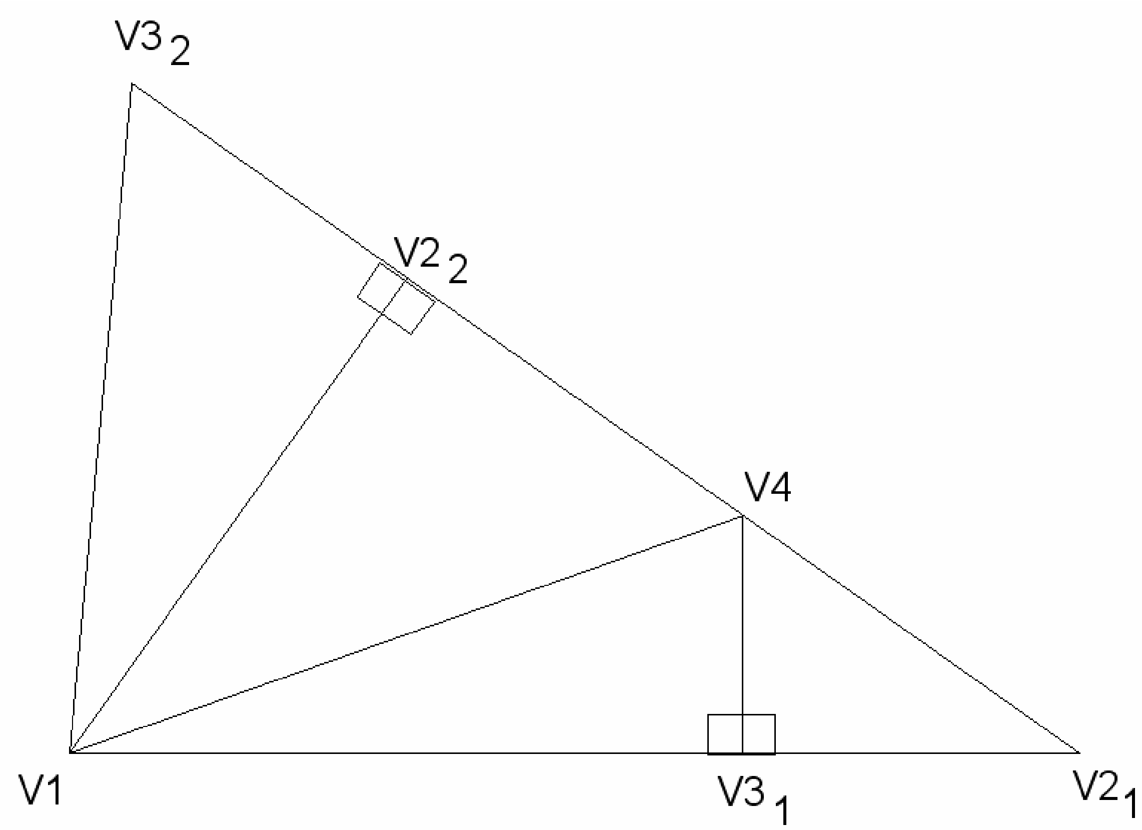

Figure 7 Layout for the A Quantum Module.

$$\alpha = 30^\circ + \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) + \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 84.735\ 611^\circ$$

$$\beta = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610^\circ$$

V1 = (0.0, 0.0) EL

$$V2_1 = \left(\frac{\sqrt{3}}{2}, 0.0\right) \text{EL} \cong (0.866\ 025\ 4, 0.0)\ \text{EL}$$

$V2_2 = (0.5 \cos(\beta), 0.5 \sin(\beta))$ EL ≅ (0.288 675 1, 0.408 248 3) EL

$$V3_1 = \left(\frac{1}{\sqrt{3}}, 0.0\right) \text{EL} \cong (0.577\ 350\ 3, 0.0)\ \text{EL}$$

$$V3_2 = \left(\frac{1}{\sqrt{3}}\cos(\alpha), \frac{1}{\sqrt{3}}\sin(\alpha)\right) \text{EL} \cong (0.052\ 972\ 9, 0.574\ 915)\ \text{EL}$$

$$V4 = \left(\frac{1}{\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) \text{EL} \cong (0.577\ 350\ 3, 0.204\ 124\ 2)\ \text{EL}$$

Comments:

The A Quantum Module is 1/6th of the 1/4-Tetrahedron. It is therefore 1/24 of the regular Tetrahedron.

There are 2 different A Quantum Modules labeled A+ and A-. These are mirror images of each other. The A+ Quantum Model can be opened and folded into the A- Quantum Model and visa versa.

The A Quantum Module does not fill all-space by itself.

The dual of the A Quantum Module is another (different) irregular Tetrahedron which is not considered further in this text.

The A Quantum Module can be subdivided into A and B Quantum Modules (6 AQM + 2 BQM).

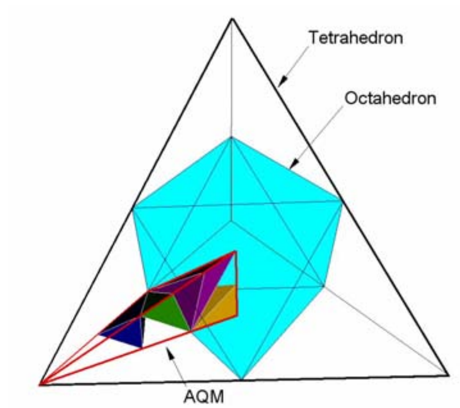

Figure 7 A Quantum Module outlined in red.

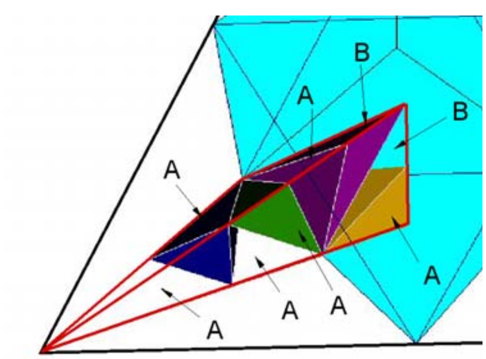

Figure 8 A Quantum Module divided into smaller A and B Quantum Modules.