Dodecahedron

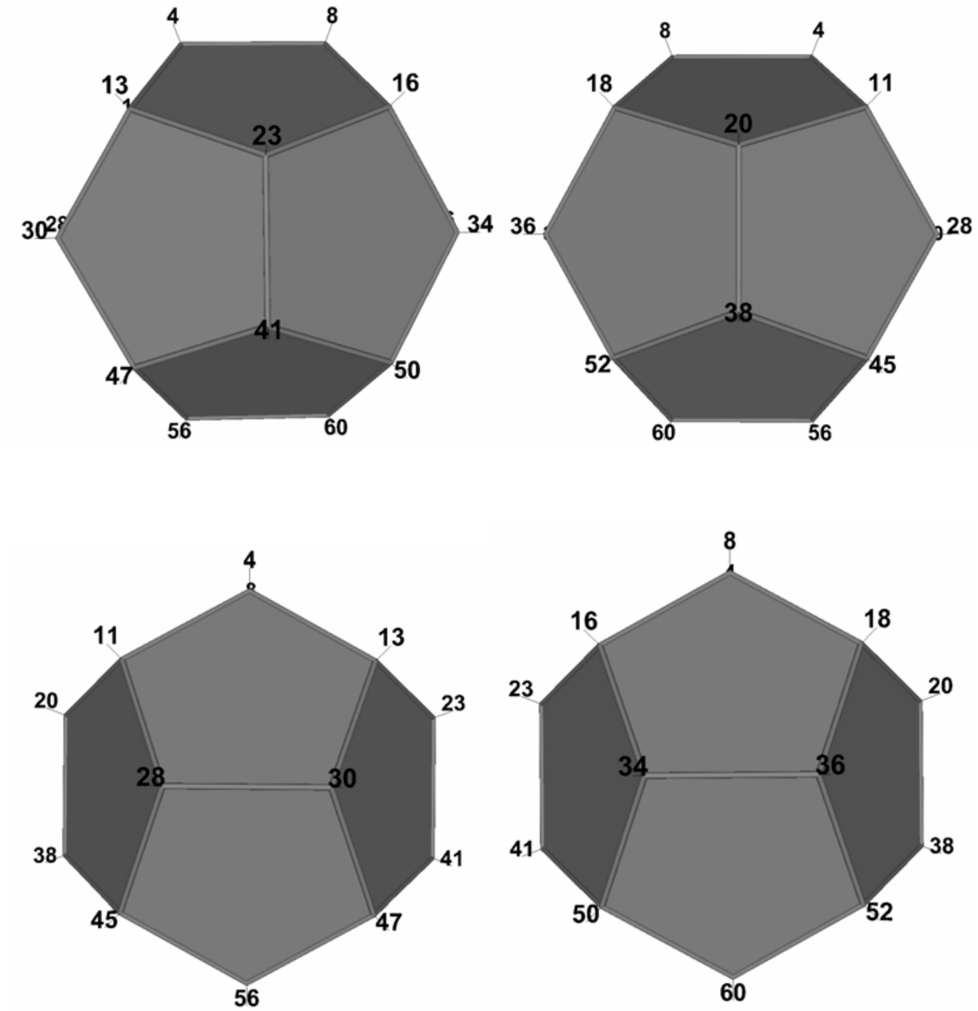

Figure 1 *Regular Dodecahedron vertex labeling using "120 Polyhedron" vertex labels.*

Topology:

Vertices = 20

Edges = 30

Faces = 12 pentagons

Lengths:

$$\varphi = \frac{1+\sqrt{5}}{2} \simeq 1.618\,033\,989$$

EL ≡ Edge length of regular Dodecahedron.

$$\text{FA} = \frac{\sqrt{4\varphi+3}}{2}\,\text{EL} \simeq 1.538\,841\,769\ \text{EL} \equiv \text{Face altitude.}$$

$$\text{DFV} = \sqrt{\frac{\varphi}{\sqrt{5}}}\ \text{EL} \cong 0.850\,650\,808\ \text{EL}$$

$$\text{DFE} = \frac{\sqrt{15+20\,\varphi}}{10}\ \text{EL} \cong 0.688\,190\,960\ \text{EL}$$

$$\text{DVV} = \frac{\sqrt{3}}{2}\,\varphi\ \text{EL} \cong 1.401\,258\,538\ \text{EL}$$

$$\text{DVE} = \frac{1}{2}\,\varphi^2\ \text{EL} \cong 1.309\,016\,994\ \text{EL}$$

$$\text{DVF} = \sqrt{\frac{7+11\,\varphi}{20}}\ \text{EL} \cong 1.113\,516\,364\ \text{EL}$$

Areas:

Area of one pentagonal face = $\frac{\sqrt{15+20\,\varphi}}{4}\,\text{EL}^2 \cong 1.720\ 477\ 401\ \text{EL}^2$

Total face area = $3\sqrt{15+20\,\varphi}\ \text{EL}^2 \cong 20.645\ 728\ 807\ \text{EL}^2$

Volume:

Cubic measure volume equation = $\frac{4+7\,\varphi}{2}\ \text{EL}^3 \cong 7.663\ 118\ 961\ \text{EL}^3$

Synergetics' Tetra-volume equation = $\frac{3\left(8+14\,\varphi\right)}{\sqrt{2}}\ \text{EL}^3 \cong 65.023\ 720\ 585\ \text{EL}^3$

Angles:

Face Angles:

All face angles are 108°.

Sum of face angles = 6480°

Central Angles:

All central angles are = $2\arcsin\left(\frac{\sqrt{3-\sqrt{5}}}{\sqrt{6}}\right) \cong 41.810\ 314\ 896°$

Dihedral Angles:

All dihedral angles are = $2\arcsin\left(\frac{2\,\varphi^2}{\sqrt{4\,\varphi+3}}\right) \cong 116.565\ 051\ 177°$

Additional Angle Information:

Note that

Central Angle(Dodecahedron) + Dihedral Angle(Icosahedron) = 180°

Central Angle(Icosahedron) + Dihedral Angle(Dodecahedron) = 180°

which is the case for dual polyhedra.

Vertex Coordinates (X, Y, Z):

The regular Dodecahedron shares its 20 vertices with that of 20 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 20 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean $\varphi = \frac{1+\sqrt{5}}{2}$. In this case, the edge length of the regular Dodecahedron is

EL = $2\,\varphi \cong$ 3.236 067 977 units of length.

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates are as follows.

V4 = $(0, \varphi, \varphi^3) \cong$ (0.0, 1.618 033 989, 4.236 067 977)
V8 = $(0, -\varphi, \varphi^3) \cong$ (0.0, -1.618 033 989, 4.236 067 977)

V11 = $(\varphi^2, \varphi^2, \varphi^2) \cong$ (2.618 033 989, 2.618 033 989, 2.618 033 989)
V13 = $(-\varphi^2, \varphi^2, \varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, 2.618 033 989)
V16 = $(-\varphi^2, -\varphi^2, \varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, 2.618 033 989)
V18 = $(\varphi^2, -\varphi^2, \varphi^2) \cong$ (2.618 033 989, -2.618 033 989, 2.618 033 989)

V20 = $(\varphi^3, 0, \varphi) \cong$ (4.236 067 977, 0.0, 1.618 033 989)
V23 = $(-\varphi^3, 0, \varphi) \cong$ (-4.236 067 977, 0.0, 1.618 033 989)
V28 = $(\varphi, \varphi^3, 0) \cong$ (1.618 033 989, 4.236 067 977, 0.0)
V30 = $(-\varphi, \varphi^3, 0) \cong$ (-1.618 033 989, 4.236 067 977, 0.0)
V34 = $(-\varphi, -\varphi^3, 0) \cong$ (-1.618 033 989, -4.236 067 977, 0.0)
V36 = $(\varphi, -\varphi^3, 0) \cong$ (1.618 033 989, -4.236 067 977, 0.0)
V38 = $(\varphi^3, 0, -\varphi) \cong$ (4.236 067 977, 0.0, -1.618 033 989)
V41 = $(-\varphi^3, 0, -\varphi) \cong$ (-4.236 067 977, 0.0, -1.618 033 989)

V45 = $(\varphi^2, \varphi^2, -\varphi^2) \cong$ (2.618 033 989, 2.618 033 989, -2.618 033 989)
V47 = $(-\varphi^2, \varphi^2, -\varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, -2.618 033 989)
V50 = $(-\varphi^2, -\varphi^2, -\varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, -2.618 033 989)
V52 = $(\varphi^2, -\varphi^2, -\varphi^2) \cong$ (2.618 033 989, -2.618 033 989, -2.618 033 989)

V56 = $(0, \varphi, -\varphi^3) \cong$ (0.0, 1.618 033 989, -4.236 067 977)
V60 = $(0, -\varphi, -\varphi^3) \cong$ (0.0, -1.618 033 989, -4.236 067 977)

Unfolded Vertex Coordinates (X, Y):

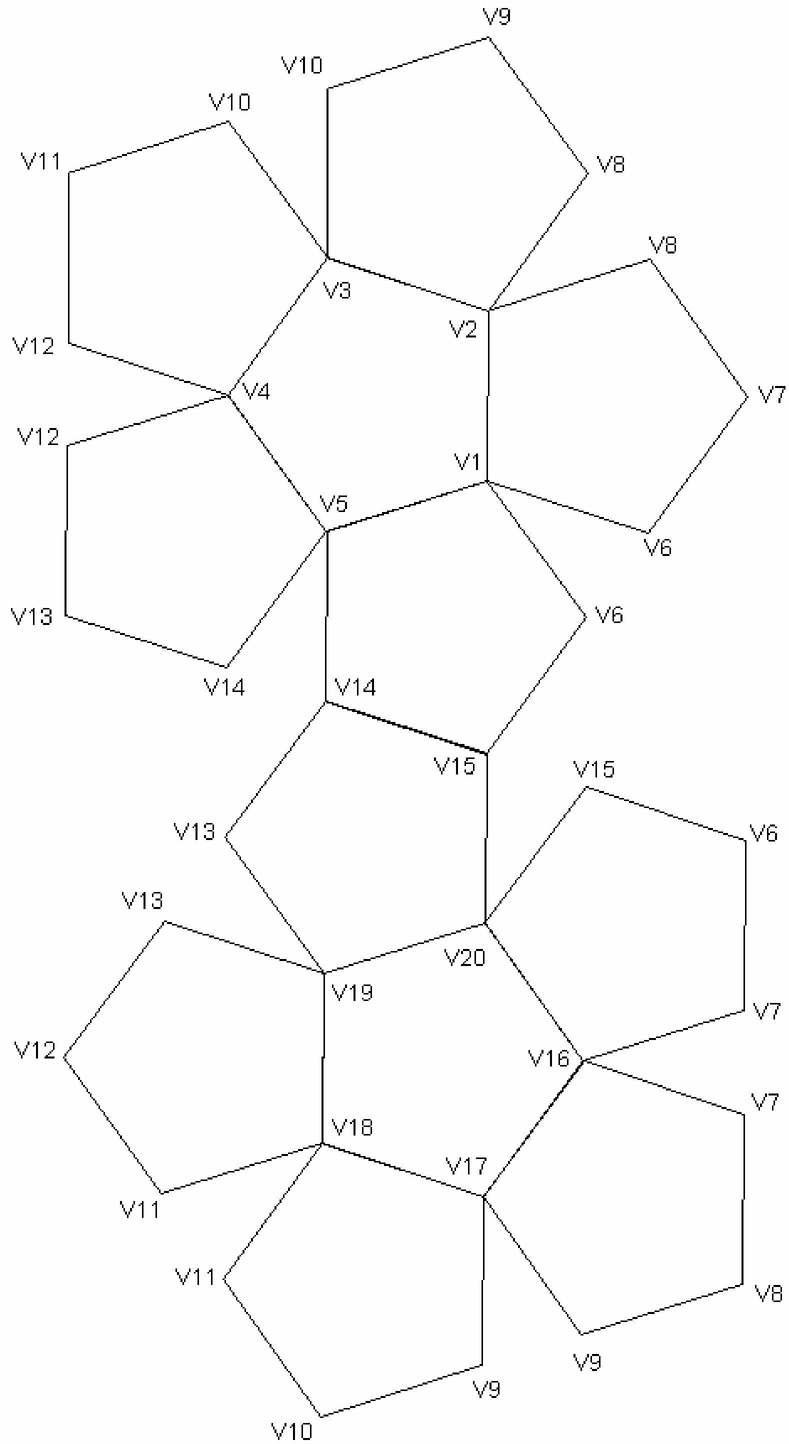

Figure 2 *Layout for the regular Dodecahedron.*

Comments:

The dual of the regular Dodecahedron is the Icosahedron.

Five intersecting Cubes share the same 20 vertices as the regular Dodecahedron.

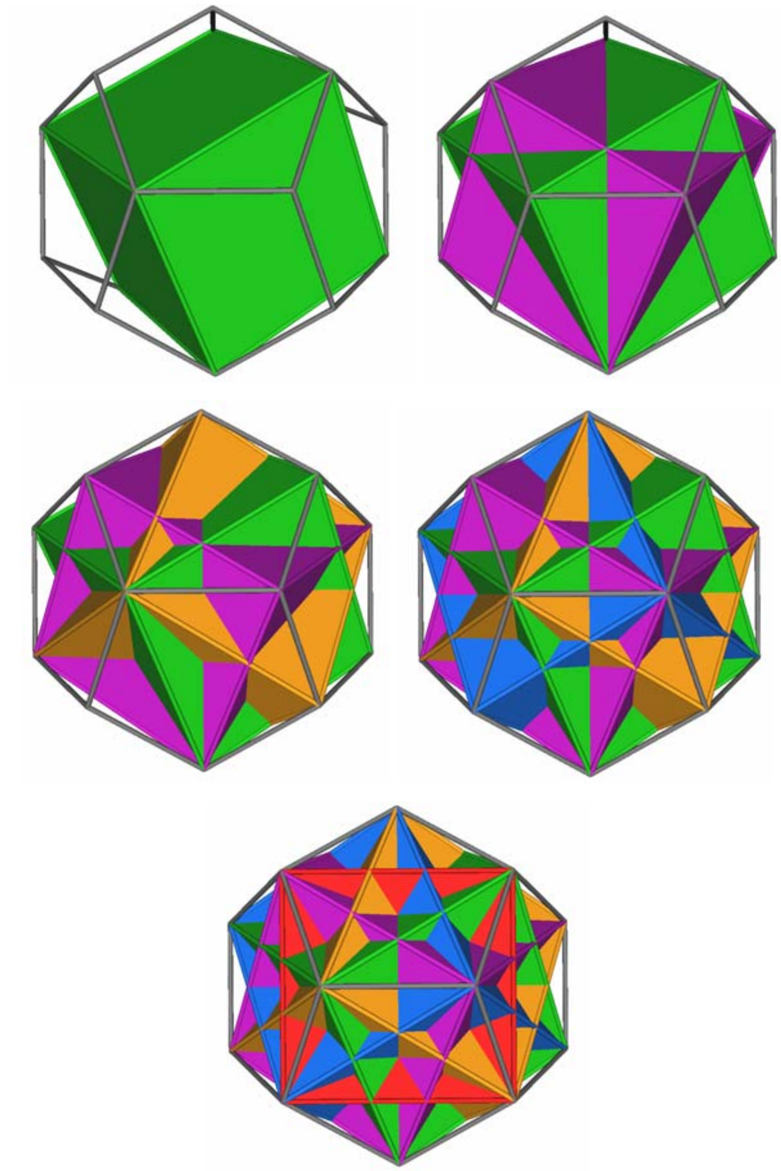

Figure 3 *Development of 5 Cubes in Dodecahedron.*

The edges of the 5 intersecting Cubes form pentagrams on each of the Dodecahedrons pentagonal faces.

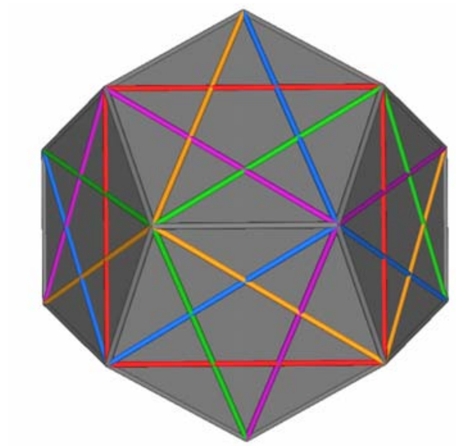

Figure 4 *Edges of 5 Cubes forms pentagrams.*

Ten intersecting regular Tetrahedra share the same 20 vertices as the regular Dodecahedron. In this case, each vertex of the Dodecahedron has 2 different Tetrahedra coincident. The Tetrahedra can be grouped into two groups of 5 Tetrahedra each such that each group of 5 Tetrahedra covers the Dodecahedron's vertices once. It then appears that the Tetrahedron's edges form a clockwise or counter clockwise orientation in the Dodecahedron.

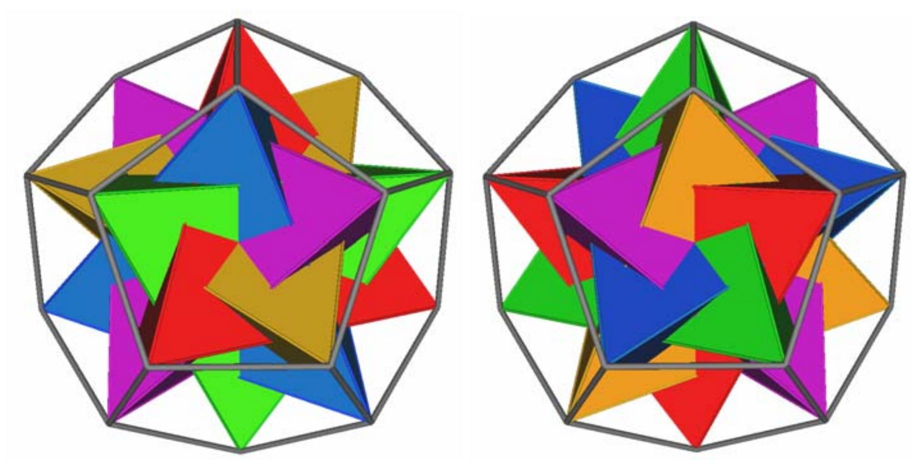

Figure 5 *Two groups of 5 intersecting Tetrahedra within Dodecahedron.*

The 5 intersecting Cubes (and 10 Tetrahedra) can be formed by considering all 5 Cubes to be initially coincident and then rotating 4 of the 5 Cubes into position. The 4 rotation axes are the 4 Vertex-to-opposite-Vertex axes of the one fixed Cube. The rotation angle is given by

$$\theta = \pm 2 \, \text{acrsin}\left(\frac{\sqrt{3}}{2\sqrt{2}\,\varphi} \right) \simeq \pm 44.477\,512^{\circ}$$

which turns out to be twice the Icosahedron "Skew Angle". The rotation directions alternate clockwise, counter clockwise (hence the "±"). For example, if rotation direction for axis 1 is clockwise then the rotation direction for axis 2 is counter clockwise, for axis 3 clockwise and for axis 4 counter clockwise.

There is a position for which the Octahedron based Jitterbug opens to share the same vertices as the regular Dodecahedron vertices.

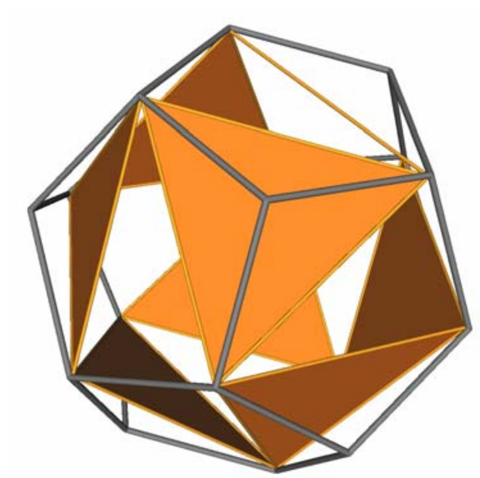

Figure 6 *Octahedron Jitterbug open to Dodecahedron position.*

As can be seen, not all of the Dodecahedron's vertices have a Jitterbug vertex coincident with it. See the data for the Jitterbug for more details.