Icosahedron

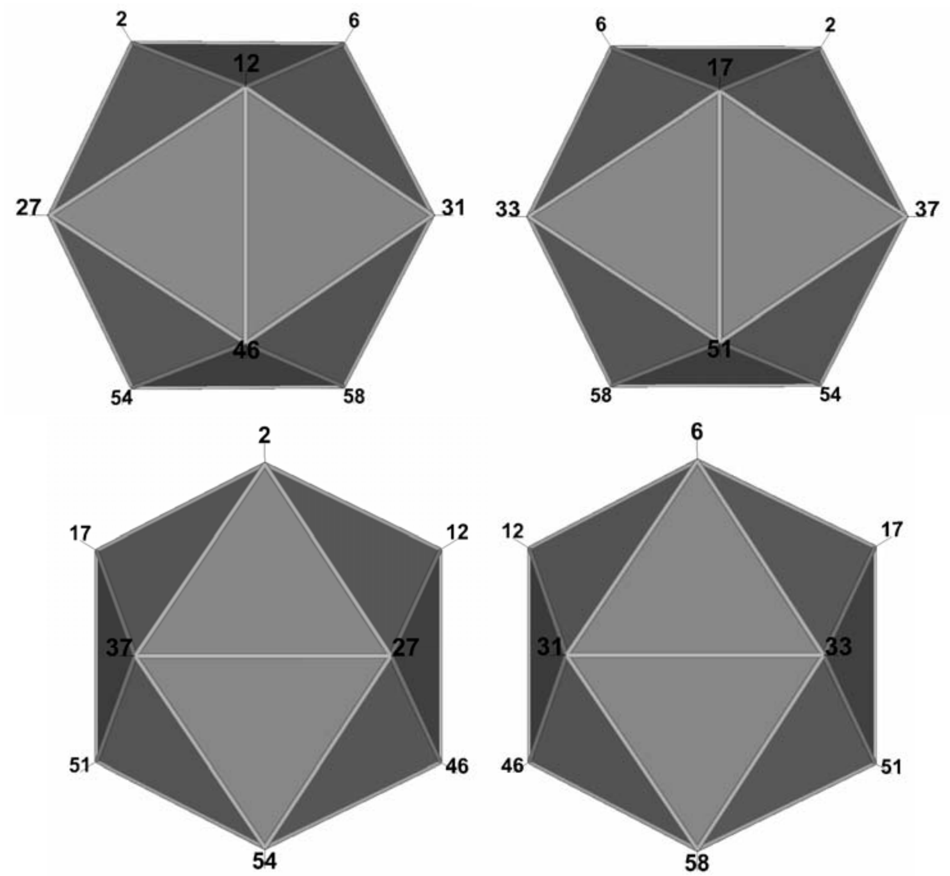

Figure 1 *Icosahedron vertex labeling using "120 Polyhedron" vertex labels.*

Topology:

Vertices = 12

Edges = 30

Faces = 20 equilateral triangles

Lengths:

$$\varphi = \frac{1+\sqrt{5}}{2} \simeq 1.618\ 033\ 989$$

EL ≡ Edge length of Icosahedron.

$$\text{FA} = \frac{\sqrt{3}}{2}\ \text{EL} \simeq 0.866\ 025\ 404\ \text{EL} \equiv \text{Face altitude.}$$

$$\text{DFV} = \frac{1}{\sqrt{3}}\ \text{EL} \cong 0.577\ 350\ 269\ \text{EL}$$

$$\text{DFE} = \frac{1}{2\sqrt{3}}\ \text{EL} \cong 0.288\ 675\ 135\ \text{EL}$$

$$\text{DVV} = \frac{\sqrt{2+\varphi}}{2}\ \text{EL} \cong 0.951\ 056\ 516\ \text{EL}$$

$$\text{DVE} = \frac{1}{2}\varphi\ \text{EL} \cong 0.809\ 016\ 994\ \text{EL}$$

$$\text{DVF} = \frac{\varphi^2}{2\sqrt{3}}\ \text{EL} \cong 0.755\ 761\ 314\ \text{EL}$$

Areas:

Area of one pentagonal face = $\frac{\sqrt{3}}{4}\,\mathrm{EL}^2 \cong 0.433\ 012\ 702\ \mathrm{EL}^2$

Total face area = $5\sqrt{3}\ \mathrm{EL}^2 \cong 8.660\ 254\ 038\ \mathrm{EL}^2$

Volume:

Cubic measure volume equation = $\frac{5\,\varphi^2}{6}\ \mathrm{EL}^3 \cong 2.181\ 694\ 991\ \mathrm{EL}^3$

Synergetics' Tetra-volume equation = $5\sqrt{2}\ \varphi^2\ \mathrm{EL}^3 \cong 18.512\ 295\ 87\ \mathrm{EL}^3$

Angles:

Face Angles:

All face angles are 60°.

Sum of face angles = 3600°

Central Angles:

All central angles are = $2\arcsin\left(\frac{\sqrt{5-\sqrt{5}}}{\sqrt{10}}\right) \cong 63.434\ 948\ 823°$

Dihedral Angles:

All dihedral angles are = $2\arcsin\left(\frac{\varphi}{\sqrt{3}}\right) \cong 138.189\ 685\ 104°$

Additional Angle Information:

Note that

Central Angle(Dodecahedron) + Dihedral Angle(Icosahedron) = 180°

Central Angle(Icosahedron) + Dihedral Angle(Dodecahedron) = 180°

which is the case for dual polyhedra.

Vertex Coordinates (X, Y, Z):

The Icosahedron shares its 12 vertices with that of 12 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 12 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean $\varphi = \frac{1+\sqrt{5}}{2}$. In this case, the edge length of the Icosahedron is

$$EL = 2\,\varphi^2 \cong 5.236\ 067\ 977 \text{ units of length.}$$

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates are as follows.

V2 = (φ^2, 0, φ^3) ≅ (2.618 033 989, 0.0, 4.236 067 977)
V6 = ($-\varphi^2$, 0, φ^3) ≅ (-2.618 033 989, 0.0, 4.236 067 977)

V12 = (0, φ^3, φ^2) ≅ (0.0, 4.236 067 977, 2.618 033 989)
V17 = (0, $-\varphi^3$, φ^2) ≅ (0.0, -4.236 067 977, 2.618 033 989)

V27 = (φ^3, φ^2, 0) ≅ (4.236 067 977, 2.618 033 989, 0.0)
V31 = ($-\varphi^3$, φ^2, 0) ≅ (-4.236 067 977, 2.618 033 989, 0.0)
V33 = ($-\varphi^3$, $-\varphi^2$, 0) ≅ (-4.236 067 977, -2.618 033 989, 0.0)
V37 = (φ^3, $-\varphi^2$, 0) ≅ (4.236 067 977, 0.0, 1.618 033 989)

V46 = (0, φ^3, $-\varphi^2$) ≅ (0.0, 4.236 067 977, -2.618 033 989)
V51 = (0, $-\varphi^3$, $-\varphi^2$) ≅ (0.0, -4.236 067 977, -2.618 033 989)

V54 = (φ^2 , 0, $-\varphi^3$) ≅ (2.618 033 989, 0.0, -4.236 067 977)
V58 = ($-\varphi^2$, 0, $-\varphi^3$) ≅ (-2.618 033 989, 0.0, -4.236 067 977)

Edge Map:

{{2, 6}, {2, 12}, {2, 17}, {2, 37}, {2, 27}, {6, 12}, {6, 17}, {6, 31},
{6, 33}, {12, 27}, {12, 46}, {12, 31}, {17, 33}, {17, 51}, {17, 37},
{27, 37}, {27, 54}, {27, 46}, {31, 46}, {31, 58}, {31, 33}, {33, 58},
{33, 51}, {37, 51}, {37, 54}, {46, 54}, {46, 58}, {51, 54}, {51, 58},
{54, 58}}

Face Map:

{{ 2, 6, 17}, { 2, 12, 6}, { 2, 17, 37}, { 2, 37, 27}, { 2, 27, 12}, {37, 54, 27}, {27, 54, 46}, {27, 46, 12}, {12, 46, 31}, {12, 31, 6}, { 6, 31, 33}, { 6, 33, 17}, {17, 33, 51}, {17, 51, 37}, {37, 51, 54}, {58, 54, 51}, {58, 46, 54}, {58, 31, 46}, {58, 33, 31}, {58, 51, 33}}

Unfolded Vertex Coordinates (X, Y):

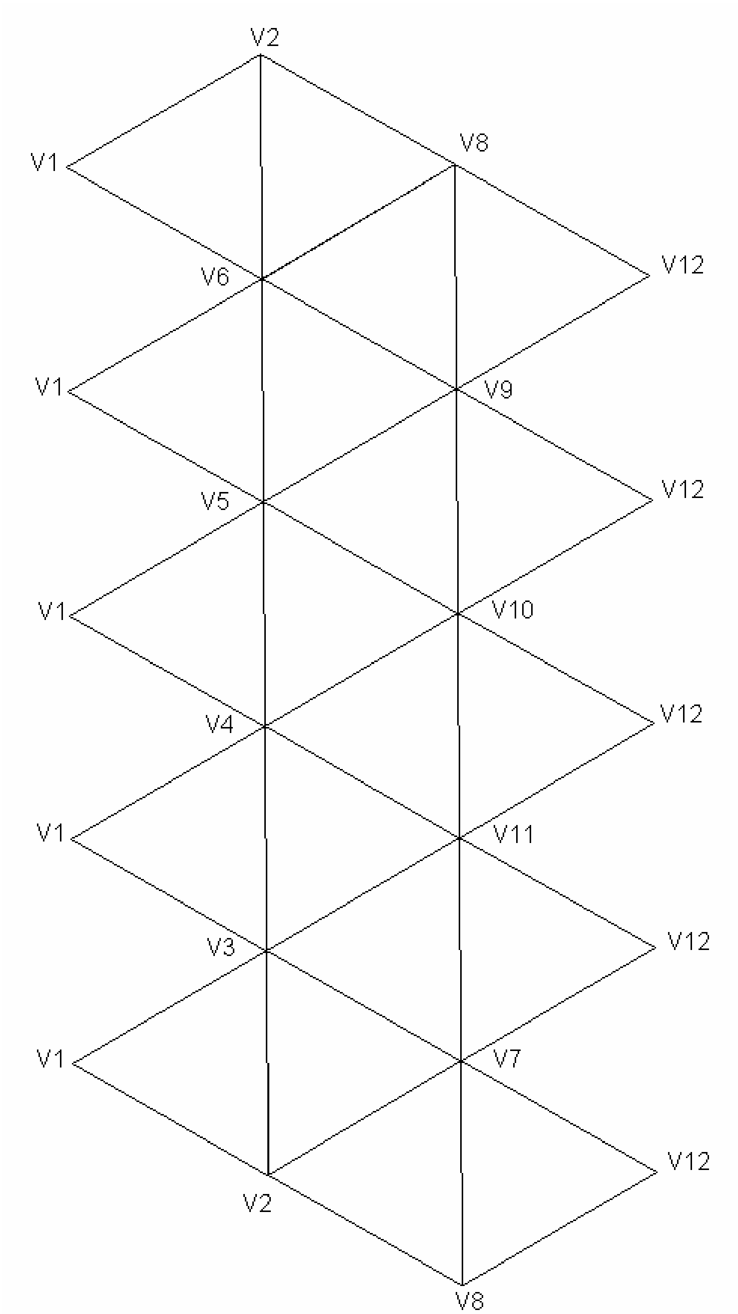

Figure 2 *Layout for the Icosahedron.*

Comments:

The dual of the Icosahedron is the regular Dodecahedron.

There is a position for which the Octahedron based Jitterbug opens to share the same vertices as the Icosahedron vertices.

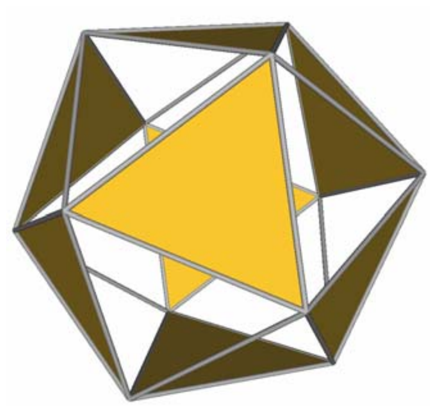

Figure 3 *Octahedron Jitterbug open to Icosahedron position.*

See the data for the Jitterbug for more details.

The 12 Icosahedron vertices can also be defined by 3 intersecting rectangular planes. The ratio of the length of the long edge to the length of the short edge is equal to the Golden Ratio.

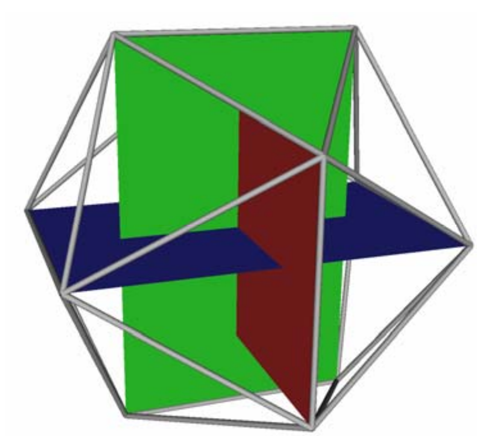

Figure 4 *Vertices defined by 3 intersecting Golden Ratio rectangles.*