Rhombic Dodecahedron

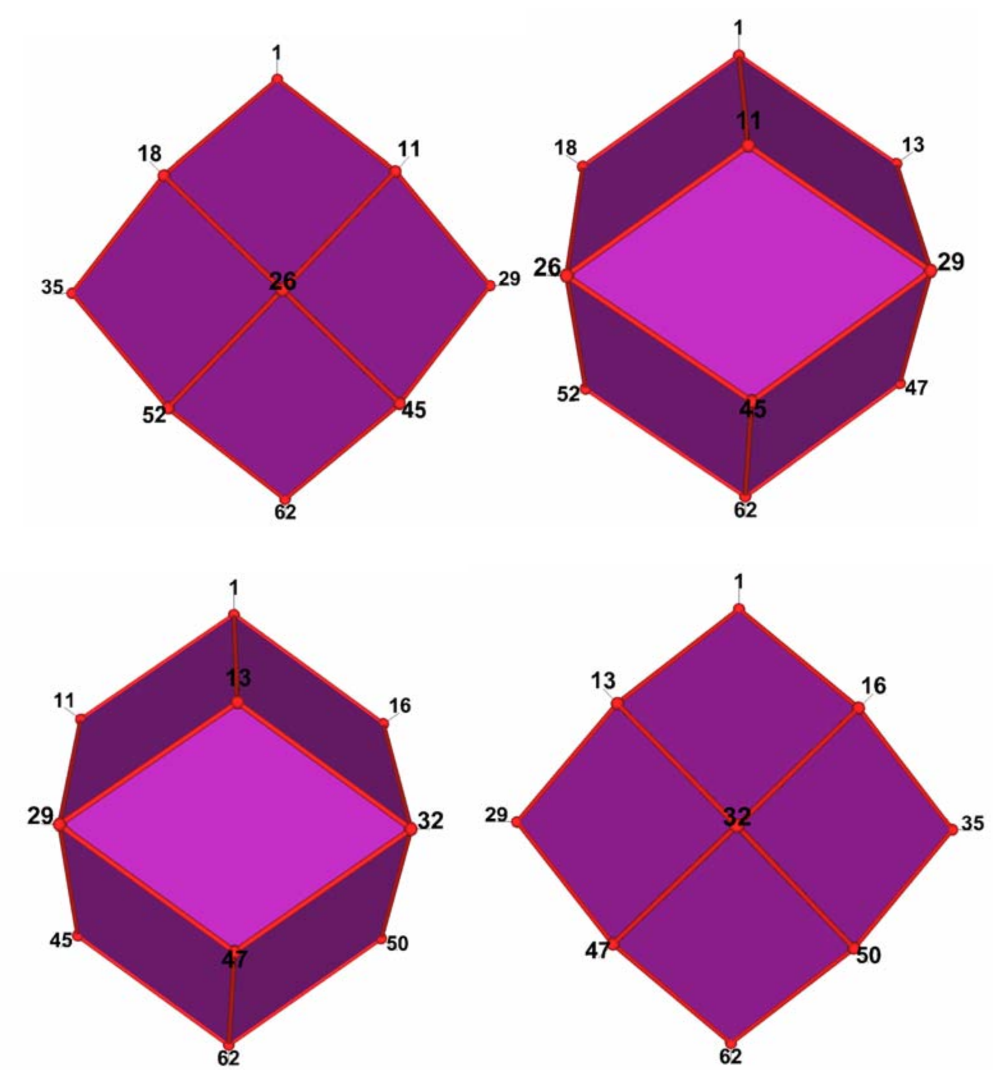

Figure 1 *Rhombic Dodecahedron.*
Vertex labels as used for the corresponding vertices of the 120 Polyhedron.

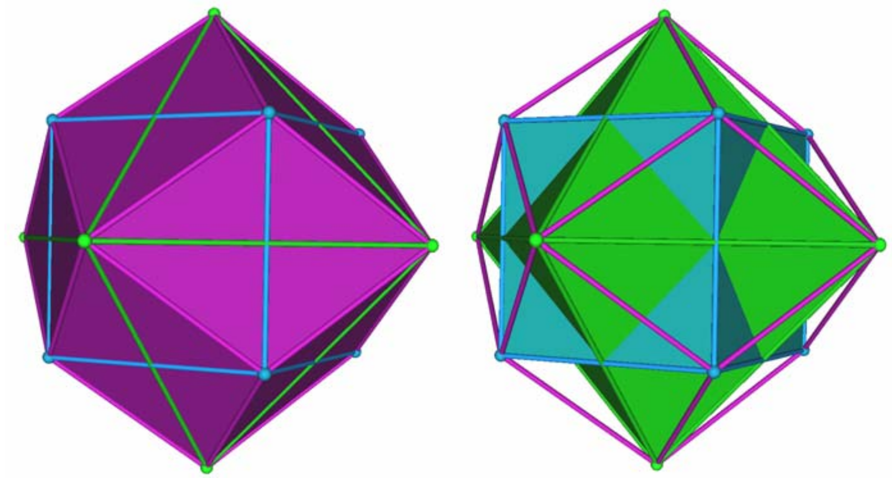

Figure 2 *Cube (blue) and Octahedron (green) define rhombic Dodecahedron.*

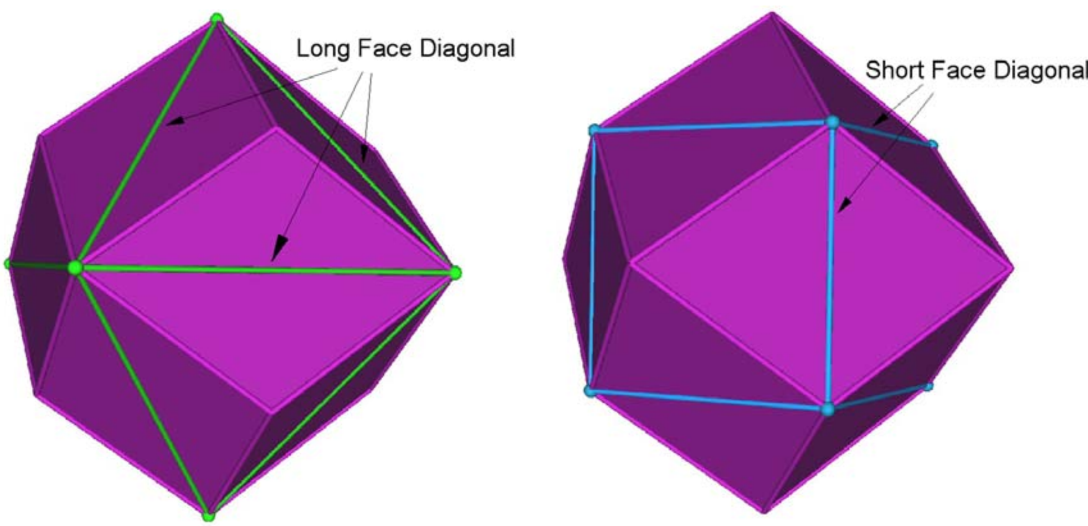

Figure 3 *"Long" (green) and "short" (blue) face diagonals.*

Topology:

Vertices = 14

Edges = 24

Faces = 12 diamonds

Lengths:

EL ≡ Edge length of rhombic Dodecahedron.

FD_L ≡ Long face diagonal = $\frac{2\sqrt{2}}{\sqrt{3}}$ EL ≅ 1.632 993 162 EL

FD_S ≡ Short face diagonal = $\frac{1}{\sqrt{2}}$ FD_L ≅ 0.707 106 781 FD_L

$FD_S = \frac{2}{\sqrt{3}}$ EL ≅ 1.154 700 538 EL

DFV_L ≡ Vertex at the end of a long face diagonal

$= \frac{1}{2} FD_L = \frac{\sqrt{2}}{\sqrt{3}}$ EL ≅ 0.816 496 581 EL

DFV_S ≡ Vertex at the end of a short face diagonal

$= \frac{1}{2\sqrt{2}} FD_L$ ≅ 0.353 553 391 FD_L

$= \frac{1}{\sqrt{3}}$ EL ≅ 0.577 350 269 EL

$$\mathrm{DFE} = \frac{\sqrt{3}}{4\sqrt{2}}\ \mathrm{FD_L} \cong 0.306\ 186\ 218\ \mathrm{FD_L}$$

$$= \frac{1}{2}\ \mathrm{EL}$$

$$\mathrm{DVV_L} = \frac{1}{\sqrt{2}}\ \mathrm{FD_L} \cong 0.707\ 106\ 781\ \mathrm{FD_L}$$

$$= \frac{2}{\sqrt{3}}\ \mathrm{EL} \cong 1.154\ 700\ 538\ \mathrm{EL}$$

$$\mathrm{DVV_S} = \frac{\sqrt{3}}{2\sqrt{2}}\ \mathrm{FD_L} \cong 0.612\ 372\ 4\ \mathrm{FD_L}$$

$$= \mathrm{EL}$$

$$\mathrm{DVE} = \frac{\sqrt{11}}{4\sqrt{2}}\ \mathrm{FD_L} \cong 0.586\ 301\ 970\ \mathrm{FD_L}$$

$$= \frac{\sqrt{11}}{2\sqrt{3}}\ \mathrm{EL} \cong 0.957\ 427\ 108\ \mathrm{EL}$$

$$\mathrm{DVF} = \frac{1}{2}\ \mathrm{FD_L}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}\ \mathrm{EL} \cong 0.816\ 496\ 581\ \mathrm{EL}$$

Areas:

Area of one diamond face = $\frac{1}{2\sqrt{2}} FD_L^2 \cong 0.353\ 553\ 391\ FD_L^2$

$= \frac{2\sqrt{2}}{3} EL^2 \cong 0.942\ 809\ 042\ EL^2$

Total face area = $3\sqrt{2}\ FD_L^2 \cong 4.242\ 640\ 687\ FD_L^2$

$= 8\sqrt{2}\ EL^2 \cong 4.242\ 640\ 687\ EL^2$

Volume:

Cubic measure volume equation = $\frac{16}{3\sqrt{3}} EL^3 \cong 3.079\ 201\ 436\ EL^3$

Synergetics' Tetra-volume equation = $6\ FD_L^3$

Angles:

Face Angles:

$\theta_S \equiv$ Face angle at short vertex = $2 \arcsin\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 109.471\ 220\ 634°$

$\theta_L \equiv$ Face angle at long vertex = $\arcsin\left(\frac{2\sqrt{2}}{3}\right) \cong 70.528\ 779\ 366°$

Sum of face angles = 4320°

Central Angles:

All central angles are = $\arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$

Dihedral Angles:

All dihedral angles are = 120°

Additional Angle Information:

Note that

Central Angle(rhombic Dodecahedron) + Dihedral Angle(VE) = 180°

Central Angle(VE) + Dihedral Angle(rhombic Dodecahedron) = 180°

which is the case for dual polyhedra.

Vertex Coordinates (X, Y, Z):

The rhombic Dodecahedron shares its 14 vertices with that of 14 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 14 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean $\varphi = \dfrac{1+\sqrt{5}}{2}$. In this case, the edge length of the rhombic Dodecahedron is

EL = $\sqrt{3(3\,\varphi + 2)}$ $\cong$ 4.534 567 884 units of length.

There are 5 intersecting rhombic Dodecahedra in the 120 Polyhedron. See Comments below.

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates for the 5 rhombic Dodecahedra are as follows.

Orientation 1:

V4 = $(0, \varphi, \varphi^3) \cong$ (0.0, 1.618 033 989, 4.236 067 977)
V7 = $(-\varphi, -\varphi^2, \varphi^3) \cong$ (-1.618 033 989, -2.618 033 989, 4.236 067 977)
V10 = $(\varphi^3, \varphi, \varphi^2) \cong$ (4.236 067 977, 1.618 033 989, 2.618 033 989)
V18 = $(\varphi^2, -\varphi^2, \varphi^2) \cong$ (2.618 033 989, -2.618 033 989, 2.618 033 989)
V22 = $(-\varphi^2, \varphi^3, \varphi) \cong$ (-2.618 033 989, 4.236 067 977, 1.618 033 989)
V23 = $(-\varphi^3, 0, \varphi) \cong$ (-4.236 067 977, 0.0, 1.618 033 989)
V28 = $(\varphi, \varphi^3, 0) \cong$ (1.618 033 989, 4.236 067 977, 0.0)
V34 = $(-\varphi, -\varphi^3, 0) \cong$ (-1.618 033 989, -4.236 067 977, 0.0)
V38 = $(\varphi^3, 0, -\varphi) \cong$ (4.236 067 977, 0.0, -1.618 033 989)
V43 = $(\varphi^2, -\varphi^3, -\varphi) \cong$ (2.618 033 989, -4.236 067 977, -1.618 033 989)
V47 = $(-\varphi^2, \varphi^2, -\varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, -2.618 033 989)
V49 = $(-\varphi^3, -\varphi, -\varphi^2) \cong$ (-4.236 067 977, -1.618 033 989, -2.618 033 989)
V55 = $(\varphi, \varphi^2, -\varphi^3) \cong$ (1.618 033 989, 2.618 033 989, -4.236 067 977)
V60 = $(0, -\varphi, -\varphi^3) \cong$ (0.0, -1.618 033 989, -4.236 067 977)

Edge Map 1:

{(7, 4), (7, 18), (7, 23), (7, 34), (10, 4), (10, 18), (10, 28), (10, 38),
(22, 4), (22, 23), (22, 28), (22, 47), (43, 18), (43, 34), (43, 38), (43, 60),
(49, 23), (49, 34), (49, 47), (49, 60), (55, 28), (55, 38), (55, 47), (55, 60)}

Orientation 2:

V4 $= (0, \varphi, \varphi^3) \cong$ (0.0, 1.618 033 989, 4.236 067 977)
V9 $= (\varphi, -\varphi^2, \varphi^3) \cong$ (1.618 033 989, -2.618 033 989, 4.236 067 977)
V14 $= (-\varphi^3, \varphi, \varphi^2) \cong$ (-4.236 067 977, 1.618 033 989, 2.618 033 989)
V16 $= (-\varphi^2, -\varphi^2, \varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, 2.618 033 989)
V20 $= (\varphi^3, 0, \varphi) \cong$ (4.236 067 977, 0.0, 1.618 033 989)
V21 $= (\varphi^2, \varphi^3, \varphi) \cong$ (2.618 033 989, 4.236 067 977, 1.618 033 989)
V30 $= (-\varphi, \varphi^3, 0) \cong$ (-1.618 033 989, 4.236 067 977, 0.0)
V36 $= (\varphi, -\varphi^3, 0) \cong$ (1.618 033 989, -4.236 067 977, 0.0)
V41 $= (-\varphi^3, 0, -\varphi) \cong$ (-4.236 067 977, 0.0, -1.618 033 989)
V42 $= (-\varphi^2, -\varphi^3, -\varphi) \cong$ (-2.618 033 989, -4.236 067 977, -1.618 033 989)
V45 $= (\varphi^2, \varphi^2, -\varphi^2) \cong$ (2.618 033 989, 2.618 033 989, -2.618 033 989)
V53 $= (\varphi^3, -\varphi, -\varphi^2) \cong$ (4.236 067 977, -1.618 033 989, -2.618 033 989)
V57 $= (-\varphi, \varphi^2, -\varphi^3) \cong$ (-1.618 033 989, 2.618 033 989, -4.236 067 977)
V60 $= (0, -\varphi, -\varphi^3) \cong$ (0.0, -1.618 033 989, -4.236 067 977)

Edge Map 2:

{(9, 4), (9, 16), (9, 20), (9, 36), (14, 4), (14, 16), (14, 30), (14, 41),
(21, 4), (21, 20), (21, 30), (21, 45), (42, 16), (42, 36), (42, 41), (42, 60),
(53, 20), (53, 36), (53, 45), (53, 60), (57, 30), (57, 41), (57, 45), (57, 60)}

Orientation 3:

V3 = $(\varphi, \varphi^2, \varphi^3) \cong$ (1.618 033 989, 2.618 033 989, 4.236 067 977)
V8 = $(0, -\varphi, \varphi^3) \cong$ (0.0, -1.618 033 989, 4.236 067 977)
V13 = $(-\varphi^2, \varphi^2, \varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, 2.618 033 989)
V15 = $(-\varphi^3, -\varphi, \varphi^2) \cong$ (-4.236 067 977, -1.618 033 989, 2.618 033 989)
V20 = $(\varphi^3, 0, \varphi) \cong$ (4.236 067 977, 0.0, 1.618 033 989)
V25 = $(\varphi^2, -\varphi^3, \varphi) \cong$ (2.618 033 989, -4.236 067 977, 1.618 033 989)
V28 = $(\varphi, \varphi^3, 0) \cong$ (1.618 033 989, 4.236 067 977, 0.0)
V34 = $(-\varphi, -\varphi^3, 0) \cong$ (-1.618 033 989, -4.236 067 977, 0.0)
V40 = $(-\varphi^2, \varphi^3, -\varphi) \cong$ (-2.618 033 989, 4.236 067 977, -1.618 033 989)
V41 = $(-\varphi^3, 0, -\varphi) \cong$ (-4.236 067 977, 0.0, -1.618 033 989)
V44 = $(\varphi^3, \varphi, -\varphi^2) \cong$ (4.236 067 977, 1.618 033 989, -2.618 033 989)
V52 = $(\varphi^2, -\varphi^2, -\varphi^2) \cong$ (2.618 033 989, -2.618 033 989, -2.618 033 989)
V56 = $(0, \varphi, -\varphi^3) \cong$ (0.0, 1.618 033 989, -4.236 067 977)
V59 = $(-\varphi, -\varphi^2, -\varphi^3) \cong$ (-1.618 033 989, -2.618 033 989, -4.236 067 977)

Edge Map 3:

{(3, 8), (3, 13), (3, 20), (3, 28), (15, 8), (15, 13), (15, 34), (15, 41),
(25, 8), (25, 20), (25, 34), (25, 52), (40, 13), (40, 28), (40, 41), (40, 56),
(44, 20), (44, 28), (44, 52), (44, 56), (59, 34), (59, 41), (59, 52), (59, 56)

Orientation 4:

V5 $= (-\varphi, \varphi^2, \varphi^3) \cong$ (-1.618 033 989, 2.618 033 989, 4.236 067 977)
V8 $= (0, -\varphi, \varphi^3) \cong$ (0.0, -1.618 033 989, 4.236 067 977)
V11 $= (\varphi^2, \varphi^2, \varphi^2) \cong$ (2.618 033 989, 2.618 033 989, 2.618 033 989)
V19 $= (\varphi^3, -\varphi, \varphi^2) \cong$ (4.236 067 977, -1.618 033 989, 2.618 033 989)
V23 $= (-\varphi^3, 0, \varphi) \cong$ (-4.236 067 977, 0.0, 1.618 033 989)
V24 $= (-\varphi^2, -\varphi^3, \varphi) \cong$ (-2.618 033 989, -4.236 067 977, 1.618 033 989)
V30 $= (-\varphi, \varphi^3, 0) \cong$ (-1.618 033 989, 4.236 067 977, 0.0)
V36 $= (\varphi, -\varphi^3, 0) \cong$ (1.618 033 989, -4.236 067 977, 0.0)
V38 $= (\varphi^3, 0, -\varphi) \cong$ (4.236 067 977, 0.0, -1.618 033 989)
V39 $= (\varphi^2, \varphi^3, -\varphi) \cong$ (2.618 033 989, 4.236 067 977, -1.618 033 989)
V48 $= (-\varphi^3, \varphi, -\varphi^2) \cong$ (-4.236 067 977, 1.618 033 989, -2.618 033 989)
V50 $= (-\varphi^2, -\varphi^2, -\varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, -2.618 033 989)
V56 $= (0, \varphi, -\varphi^3) \cong$ (0.0, 1.618 033 989, -4.236 067 977)
V61 $= (\varphi, -\varphi^2, -\varphi^3) \cong$ (1.618 033 989, -2.618 033 989, -4.236 067 977)

Edge Map 4:

{(5, 8), (5, 11), (5, 23), (5, 30), (19, 8), (19, 11), (19, 36), (19, 38),
(24, 8), (24, 23), (24, 36), (24, 50), (39, 11), (39, 30), (39, 38), (39, 56),
(48, 23), (48, 30), (48, 50), (48, 56), (61, 36), (61, 38), (61, 50), (61, 56)}

Orientation 5:

V1 = $(0, 0, 2\varphi^2) \cong$ (0.0, 0.0, 5.236 067 977)
V11 = $(\varphi^2, \varphi^2, \varphi^2) \cong$ (2.618 033 989, 2.618 033 989, 2.618 033 989)
V13 = $(-\varphi^2, \varphi^2, \varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, 2.618 033 989)
V16 = $(-\varphi^2, -\varphi^2, \varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, 2.618 033 989)
V18 = $(\varphi^2, -\varphi^2, \varphi^2) \cong$ (2.618 033 989, -2.618 033 989, 2.618 033 989)
V26 = $(2\varphi^2, 0, 0) \cong$ (5.236 067 977, 0.0, 0.0)
V29 = $(0, 2\varphi^2, 0) \cong$ (0.0, 5.236 067 977, 0.0)
V32 = $(-2\varphi^2, 0, 0) \cong$ (-5.236 067 977, 0.0, 0.0)
V35 = $(0, -2\varphi^2, 0) \cong$ (0.0, -5.236 067 977, 0.0)
V45 = $(\varphi^2, \varphi^2, -\varphi^2) \cong$ (2.618 033 989, 2.618 033 989, -2.618 033 989)
V47 = $(-\varphi^2, \varphi^2, -\varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, -2.618 033 989)
V50 = $(-\varphi^2, -\varphi^2, -\varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, -2.618 033 989)
V52 = $(\varphi^2, -\varphi^2, -\varphi^2) \cong$ (2.618 033 989, -2.618 033 989, -2.618 033 989)
V62 = $(0, 0, -2\varphi^2) \cong$ (0.0, 0.0, -5.236 067 977)

Edge Map 5:

{(1, 11), (1, 13), (1, 16), (1, 18), (26, 18), (26, 11), (26, 45), (26, 52),
(29, 11), (29, 13), (29, 47), (29, 45), (32, 13), (32, 16), (32, 50), (32, 47),
(35, 16), (35, 18), (35, 52), (35, 50), (62, 45), (62, 47), (62, 50), (62, 52)}

Unfolded Vertex Coordinates (X, Y):

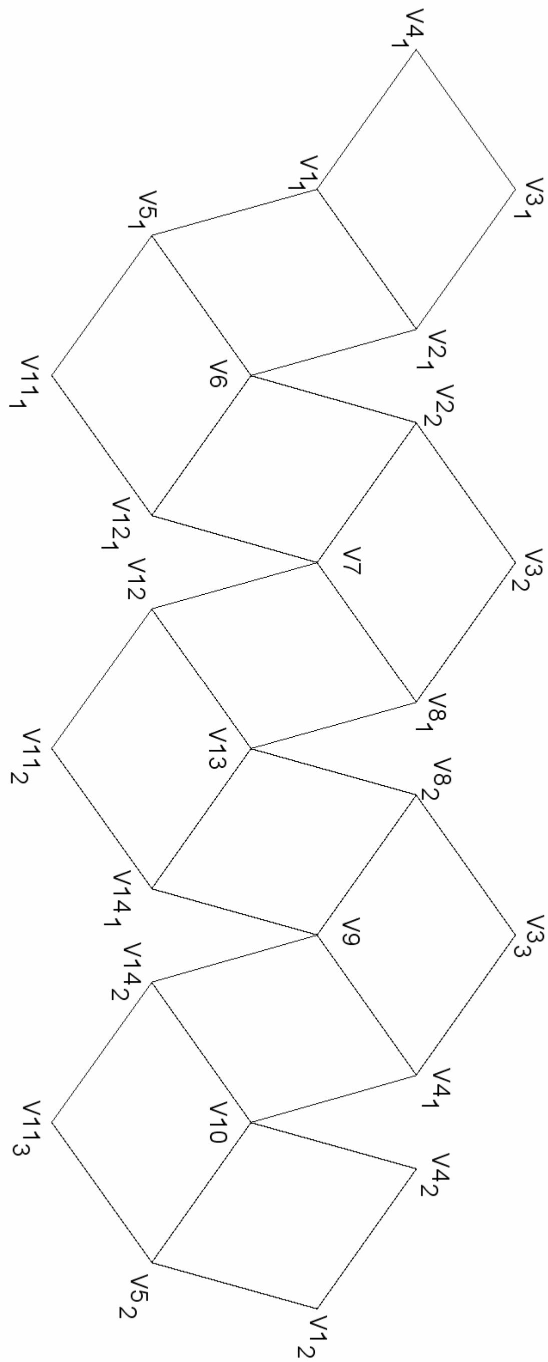

Figure 4 *Layout for the rhombic Dodecahedron.*

Comments:

Five intersecting rhombic Dodecahedra share vertices with the 120 Polyhedron.

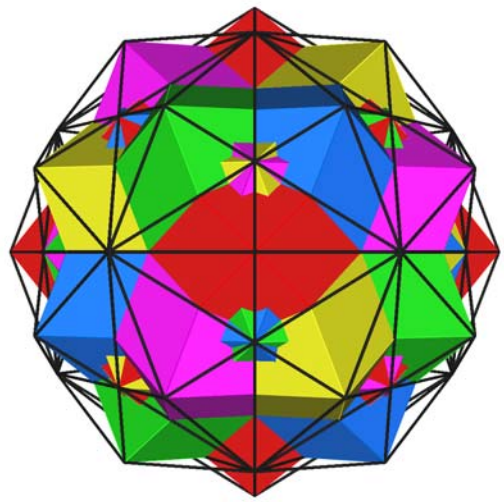

Figure 5 *Five intersecting rhombic dodecahedra in the 120 Polyhedron.*

Each of the 5 intersecting rhombic Dodecahedra has a Cube defined by the short face diagonals. The orientation of the 5 intersecting rhombic Dodecahedra can be formed by considering all 5 rhombic Dodecahedra (and the associated Cubes) to be initially coincident and then rotating 4 of the 5 rhombic Dodecahedra (and Cubes) into position. The 4 rotation axes are the 4 Vertex-to-opposite-Vertex axes of the one fixed Cube of the one fixed (not rotated) rhombic Dodecahedron. The rotation angle is given by

$$\theta = \pm 2 \text{ acrsin}\left(\frac{\sqrt{3}}{2\sqrt{2}\ \varphi} \right) \simeq \pm 44.477\,512^{\circ}$$

The rotation directions alternate clockwise, counter clockwise (hence the "$\pm$") for alternate axes. For example, if rotation direction for axis 1 is clockwise then the rotation direction for axis 2 is counter clockwise, for axis 3 clockwise and for axis 4 counter clockwise.