Rhombic Triacontahedron

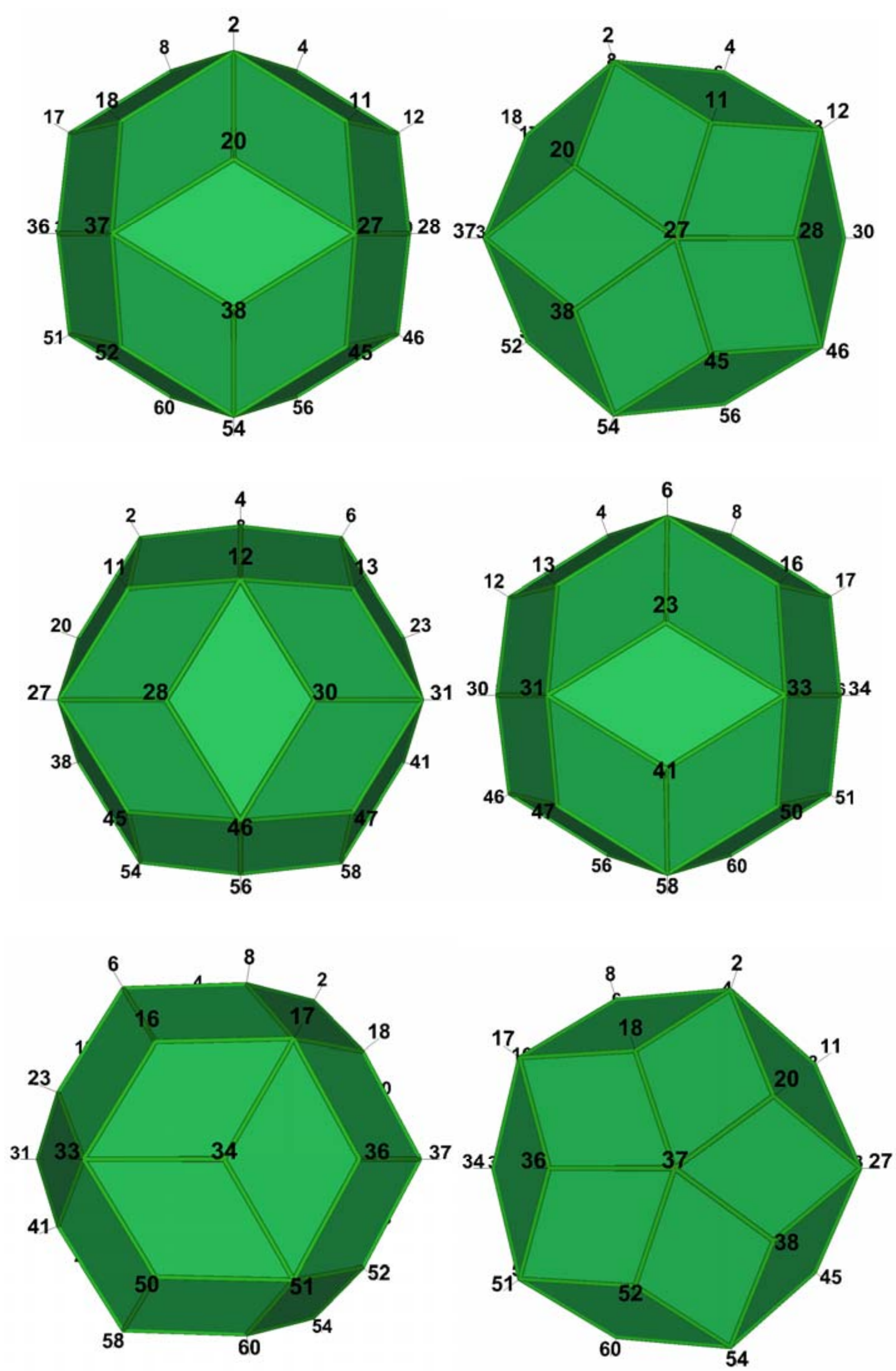

Figure 1 *Rhombic Triacontahedron.*
Vertex labels as used for the corresponding vertices of the 120 Polyhedron.

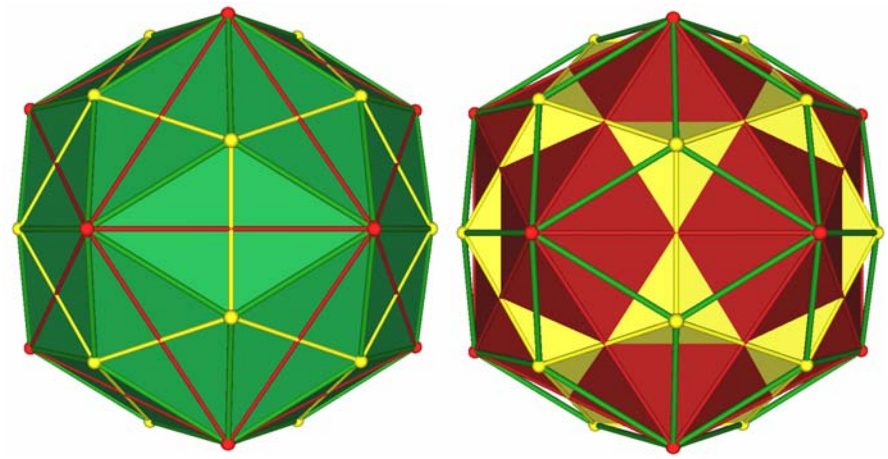

Figure 2 *Icosahedron (red) and Dodecahedron (yellow) define the rhombic Triacontahedron (green).*

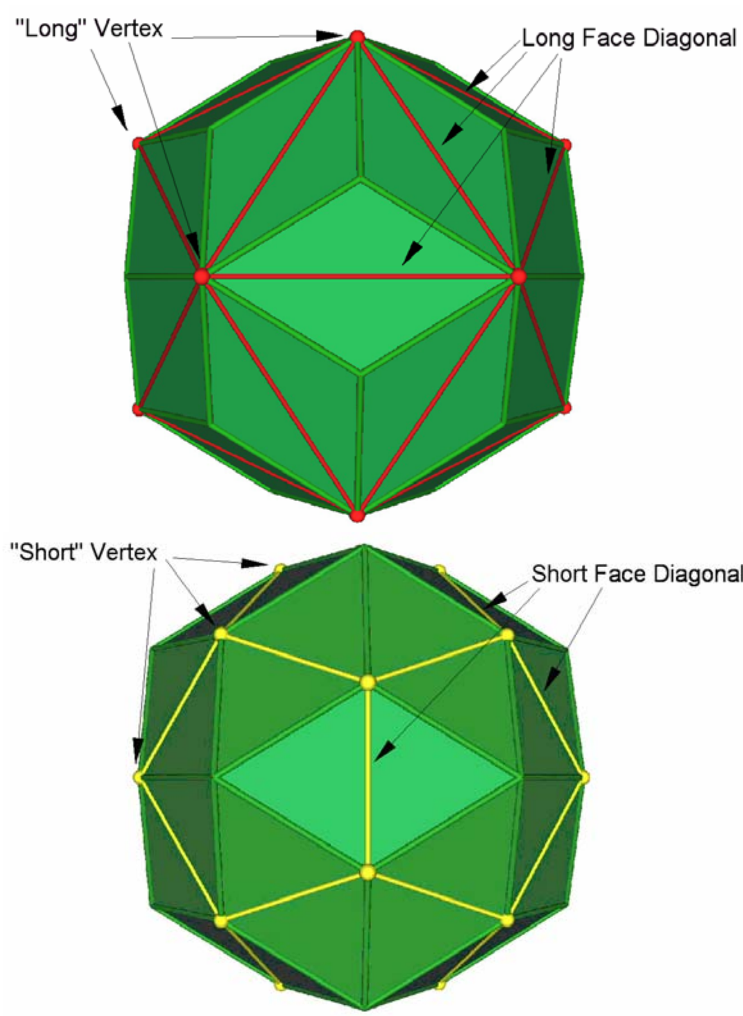

Figure 3 *"Long" (red) and "short" (yellow) face diagonals and vertices.*

Topology:

Vertices = 32

Edges = 60

Faces = 30 diamonds

Lengths:

$$\varphi = \frac{1+\sqrt{5}}{2}$$

EL ≡ Edge length of rhombic Triacontahedron.

$$EL = \frac{\sqrt{\varphi+2}}{2\varphi} FD_L \cong 0.587\ 785\ 252\ FD_L$$

$$= \frac{\sqrt{3-\varphi}}{\varphi}\ DVF$$

$$FD_L \equiv \text{Long face diagonal} = \frac{2}{\varphi} DVF \cong 1.236\ 067\ 977\ DVF$$

$$= \frac{2}{\sqrt{3-\varphi}}\ EL \cong 1.701\ 301\ 617\ EL$$

$$FD_S \equiv \text{Short face diagonal} = \frac{1}{\varphi}\ FD_L \cong 0.618\ 033\ 989\ FD_L$$

$$= \frac{2}{\varphi^2}\ DVF \cong 0.763\ 932\ 023\ DVF$$

$$= \frac{2}{\sqrt{\varphi+2}}\ EL \cong 1.051\ 462\ 224\ EL$$

$\mathrm{DFV_L}$ ≡ Center of face to vertex at the end of a long face diagonal

$$= \frac{1}{2}\ \mathrm{FD_L}$$

$$= \frac{1}{\varphi}\ \mathrm{DVF} \cong\ 0.618\ 033\ 989\ \mathrm{DVF}$$

$$= \frac{1}{\sqrt{3-\varphi}}\ \mathrm{EL} \cong\ 0.850\ 650\ 808\ \mathrm{EL}$$

$\mathrm{DFV_S}$ ≡ Center of face to vertex at the end of a short face diagonal

$$= \frac{1}{2\,\varphi}\ \mathrm{FD_L}\ \cong\ 0.309\ 016\ 994\ \mathrm{FD_L}$$

$$= \frac{1}{\varphi^2}\ \mathrm{DVF} \cong\ 0.381\ 966\ 011\ \mathrm{DVF}$$

$$= \frac{1}{\sqrt{\varphi+2}}\ \mathrm{EL} \cong\ 0.525\ 731\ 112\ \mathrm{EL}$$

$$\mathrm{DFE} = \frac{\sqrt{\varphi+2}}{4\,\varphi}\,\mathrm{FD_L} \cong 0.293\ 892\ 626\ \mathrm{FD_L}$$

$$= \frac{\sqrt{\varphi+2}}{2\left(\varphi+1\right)}\,\mathrm{DVF} \cong 0.363\ 271\ 264\ \mathrm{DVF}$$

$$= \frac{1}{2}\,\mathrm{EL}$$

$$\mathrm{DVV_L} = \frac{\varphi\sqrt{3-\varphi}}{2}\,\mathrm{FD_L} \cong 0.951\ 056\ 516\ \mathrm{FD_L}$$

$$= \sqrt{3-\varphi}\ \ \mathrm{DVF}\ \cong 1.175\ 570\ 505\ \mathrm{DVF}$$

$$= \varphi\ \ \mathrm{EL} \cong 1.618\ 033\ 988\ \mathrm{EL}$$

$$\mathrm{DVV_S} = \frac{\sqrt{3}}{2}\ \mathrm{FD_L} \cong 0.866\ 025\ 403\ \mathrm{FD_L}$$

$$= \frac{\sqrt{3}}{\varphi}\ \mathrm{DVF} \cong 1.070\ 466\ 269\ \mathrm{DVF}$$

$$= \frac{\sqrt{3}}{\sqrt{3-\varphi}}\ \mathrm{EL} \cong 1.473\ 370\ 419\ \mathrm{EL}$$

$$\mathrm{DVE} = \frac{\sqrt{17+3\sqrt{5}}}{4\sqrt{2}}\ \mathrm{FD_L} \cong 0.860\ 744\ 662\ \mathrm{FD_L}$$

$$= \frac{\sqrt{9-2\sqrt{5}}}{2}\ \mathrm{DVF} \cong 1.063\ 938\ 913\ \mathrm{DVF}$$

$$= \frac{\sqrt{25+8\sqrt{5}}}{2\sqrt{5}}\ \mathrm{EL} \cong 1.464\ 386\ 285\ \mathrm{EL}$$

$$\mathrm{DVF} = \frac{1}{2\varphi}\ \mathrm{FD_L} \cong 0.809\ 016\ 994\ \mathrm{FD_L}$$

$$= \frac{1}{\varphi^2}\ \mathrm{DVF} \cong 0.381\ 966\ 011\ \mathrm{DVF}$$

$$= \frac{\varphi}{\sqrt{3-\varphi}}\ \mathrm{EL} \cong 1.376\ 381\ 920\ \mathrm{EL}$$

Areas:

Area of one diamond face $= \frac{1}{2\varphi} FD_L^{\,2} \cong 0.309\,016\,994\ FD_L^{\,2}$

$$= \frac{2}{\varphi^3} DVF^{\,2} \cong 0.472\,136\,955\ DVF^{\,2}$$

$$= \frac{2}{2\varphi - 1} EL^{\,2} \cong 0.894\,427\,191\ EL^{\,2}$$

Total face area $= \frac{15}{\varphi} FD_L^{\,2} \cong 9.270\,509\,831\ FD_L^{\,2}$

$$= \frac{60}{\varphi^3} DVF^{\,2} \cong 14.164\,078\,650\ DVF^{\,2}$$

$$= \frac{60}{2\varphi - 1} EL^{\,2} \cong 26.832\,815\,730\ EL^{\,2}$$

Volume:

Cubic measure volume equation $= \frac{5}{2} FD_L^{\,3} = 2.5\ FD_L^{\,3}$

$$= \frac{20}{\varphi^3} DVF^{\,3} \cong 4.721\,359\,550\ DVF^{\,3}$$

$$= \frac{20}{(3-\varphi)\sqrt{3-\varphi}} EL^{\,3} \cong 12.310\,734\,149\ EL^{\,3}$$

Synergetics' Tetra-volume equation $= 15\sqrt{2}\ FD_L^{\,3} \cong 21.213\,203\,436\ FD_L^{\,3}$

$$= 120\sqrt{18 - 8\sqrt{5}}\ DVF^{\,3} \cong 40.062\,064\,251\ DVF^{\,3}$$

$$= \left(60 + 12\sqrt{5}\right)\sqrt{\frac{5+\sqrt{5}}{5}}\ EL^{\,3} \cong 104.460\,043\,175\ EL^{\,3}$$

Angles:

Face Angles:

$\theta_S \equiv$ Face angle at short vertex $= 2 \arccos\left(\dfrac{\sqrt{10-2\sqrt{5}}}{2\sqrt{5}}\right) \cong 116.565\ 051\ 177°$

$\theta_L \equiv$ Face angle at long vertex $= 2 \arccos\left(\dfrac{\sqrt{5+\sqrt{5}}}{\sqrt{10}}\right) \cong 63.434\ 948\ 823°$

Sum of face angles = 10800°

Central Angles:

All central angles are $= \arccos\left(\dfrac{\varphi}{\sqrt{9-3\varphi}}\right) \cong 37.377\ 368\ 141°$

Dihedral Angles:

All dihedral angles are = 144°

Vertex Coordinates (X, Y, Z):

The rhombic Triacontahedron shares its 32 vertices with that of 32 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 32 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean $\varphi = \frac{1+\sqrt{5}}{2}$. In this case, the edge length of the rhombic Triacontahedron is

EL = $\varphi\sqrt{\varphi + 2} \cong 3.077\ 683\ 537$ units of length.

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates for the rhombic Triacontahedron are as follows.

V2 = $(\varphi^2, 0, \varphi^3) \cong$ (2.618 033 989, 0.0, 4.236 067 977)
V4 = $(0, \varphi, \varphi^3) \cong$ (0.0, 1.618 033 989, 4.236 067 977)
V6 = $(-\varphi^2, 0, \varphi^3) \cong$ (-2.618 033 989, 0.0, 4.236 067 977)
V8 = $(0, -\varphi, \varphi^3) \cong$ (0.0, -1.618 033 989, 4.236 067 977)
V11 = $(\varphi^2, \varphi^2, \varphi^2) \cong$ (2.618 033 989, 2.618 033 989, 2.618 033 989)
V12 = $(0, \varphi^3, \varphi^2) \cong$ (0.0, 4.236 067 977, 2.618 033 989)
V13 = $(-\varphi^2, \varphi^2, \varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, 2.618 033 989)
V16 = $(-\varphi^2, -\varphi^2, \varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, 2.618 033 989)
V17 = $(0, -\varphi^3, \varphi^2) \cong$ (0.0, -4.236 067 977, 2.618 033 989)
V18 = $(\varphi^2, -\varphi^2, \varphi^2) \cong$ (2.618 033 989, -2.618 033 989, 2.618 033 989)
V20 = $(\varphi^3, 0, \varphi) \cong$ (4.236 067 977, 0.0, 1.618 033 989)
V23 = $(-\varphi^3, 0, \varphi) \cong$ (-4.236 067 977, 0.0, 1.618 033 989)
V27 = $(\varphi^3, \varphi^2, 0) \cong$ (4.236 067 977, 2.618 033 989, 0.0)
V28 = $(\varphi, \varphi^3, 0) \cong$ (1.618 033 989, 4.236 067 977, 0.0)
V30 = $(-\varphi, \varphi^3, 0) \cong$ (-1.618 033 989, 4.236 067 977, 0.0)
V31 = $(-\varphi^3, \varphi^2, 0) \cong$ (-4.236 067 977, 2.618 033 989, 0.0)
V33 = $(-\varphi^3, -\varphi^2, 0) \cong$ (-4.236 067 977, -2.618 033 989, 0.0)
V34 = $(-\varphi, -\varphi^3, 0) \cong$ (-1.618 033 989, -4.236 067 977, 0.0)
V36 = $(\varphi, -\varphi^3, 0) \cong$ (1.618 033 989, -4.236 067 977, 0.0)
V37 = $(\varphi^3, -\varphi^2, 0) \cong$ (4.236 067 977, -2.618 033 989, 0.0)
V38 = $(\varphi^3, 0, -\varphi) \cong$ (4.236 067 977, 0.0, -1.618 033 989)
V41 = $(-\varphi^3, 0, -\varphi) \cong$ (-4.236 067 977, 0.0, -1.618 033 989)
V45 = $(\varphi^2, \varphi^2, -\varphi^2) \cong$ (2.618 033 989, 2.618 033 989, -2.618 033 989)
V46 = $(0, \varphi^3, -\varphi^2) \cong$ (0.0, 4.236 067 977, -2.618 033 989)
V47 = $(-\varphi^2, \varphi^2, -\varphi^2) \cong$ (-2.618 033 989, 2.618 033 989, -2.618 033 989)
V50 = $(-\varphi^2, -\varphi^2, -\varphi^2) \cong$ (-2.618 033 989, -2.618 033 989, -2.618 033 989)
V51 = $(0, -\varphi^3, -\varphi^2) \cong$ (0.0, -4.236 067 977, -2.618 033 989)
V52 = $(\varphi^2, -\varphi^2, -\varphi^2) \cong$ (2.618 033 989, -2.618 033 989, -2.618 033 989)
V54 = $(\varphi^2, 0, -\varphi^3) \cong$ (2.618 033 989, 0.0, -4.236 067 977)
V56 = $(0, \varphi, -\varphi^3) \cong$ (0.0, 1.618 033 989, -4.236 067 977)
V58 = $(-\varphi^2, 0, -\varphi^3) \cong$ (-2.618 033 989, 0.0, -4.236 067 977)
V60 = $(0, -\varphi, -\varphi^3) \cong$ (0.0, -1.618 033 989, -4.236 067 977)

Edge Map:

{ (2, 4), (4, 6), (6, 8), (8, 2), (2, 11), (11, 12), (4, 12),
(12, 13), (13, 6), (6, 23), (6, 16), (16, 17), (17, 8), (17, 18), (2, 18),
(2, 20), (20, 27), (27, 28), (12, 28), (12, 30), (13, 31), (23, 31), (23, 33),
(33, 16), (18, 37), (37, 20), (11, 27), (54, 56), (56, 58), (58, 60), (60, 54),
(54, 45), (45, 46), (46, 56), (58, 47), (58, 41), (58, 50), (60, 51), (52, 54),
(54, 38), (38, 27), (27, 45), (46, 47), (47, 31), (31, 41), (41, 33), (33, 50),
(50, 51), (51, 52), (52, 37), (37, 38), (28, 46), (30, 46), (30, 31), (17, 36),
(36, 51), (51, 34), (34, 17), (36, 37), (33, 34)}

Comments:

The central angle of the rhombic Triacontahedron, which is

$$\arccos\left(\frac{\varphi}{\sqrt{9-3\varphi}}\right) \cong 37.377\ 368\ 141°$$

is also the angular amount that the Jitterbug vertex travels around the vertex path ellipse to go from the Dodecahedron position to the Icosahedron position.

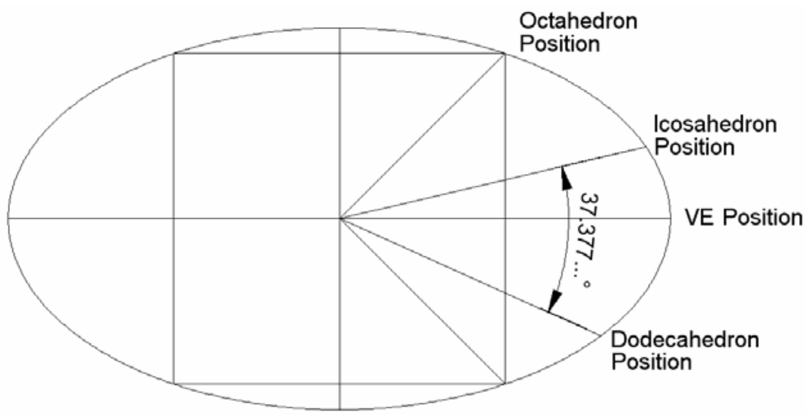

Figure 4 *Elliptical path of a Jitterbug vertex showing Icosahedron and Dodecahedron positions.*